EXAMPLE 5.1

Draw the free-body diagram of the uniform beam shown in Fig. 5–7a. The beam has a mass of 100 kg.
The free-body diagram of the beam is shown in Fig. 5–7b. Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as \( A_x \), \( A_y \), and \( M_A \). The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, \( W = 100(9.81) \) N = 981 N, acts through the beam’s center of gravity \( G \), which is 3 m from A since the beam is uniform.
Draw the free-body diagram of the foot lever shown in Fig. 5–8a. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb.

Fig. 5–8
EXAMPLE 5.2 CONTINUED

SOLUTION

By inspection of the photo the lever is loosely bolted to the frame at A. The rod at B is pinned at its ends and acts as a “short link.” After making the proper measurements, the idealized model of the lever is shown in Fig. 5–8b. From this, the free-body diagram is shown in Fig. 5–8c. The pin support at A exerts force components $A_x$ and $A_y$ on the lever. The link at B exerts a force of 20 lb, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be $k = 20 \text{ lb/in.}$, then since the stretch $s = 1.5 \text{ in.}$, using Eq. 3–2, $F_s = ks = 20 \text{ lb/in.} \times 1.5 \text{ in.} = 30 \text{ lb}$. Finally, the operator’s shoe applies a vertical force of $F$ on the pedal. The dimensions of the lever are also shown on the free-body diagram, since this information will be useful when computing the moments of the forces. As usual, the senses of the unknown forces at A have been assumed. The correct senses will become apparent after solving the equilibrium equations.
Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. 5–9a. Draw the free-body diagrams for each pipe and both pipes together.
EXAMPLE | 5.3 CONTINUED

SOLUTION

The idealized model from which we must draw the free-body diagrams is shown in Fig. 5–9b. Here the pipes are identified, the dimensions have been added, and the physical situation reduced to its simplest form.

The free-body diagram for pipe A is shown in Fig. 5–9c. Its weight is \( W = 300(9.81) \) N = 2943 N. Assuming all contacting surfaces are smooth, the reactive forces \( T, F, R \) act in a direction normal to the tangent at their surfaces of contact.

The free-body diagram of pipe B is shown in Fig. 5–9d. Can you identify each of the three forces acting on this pipe? In particular, note that \( R \), representing the force of \( A \) on \( B \), Fig. 5–9d, is equal and opposite to \( R \) representing the force of \( B \) on \( A \), Fig. 5–9c. This is a consequence of Newton’s third law of motion.

The free-body diagram of both pipes combined (“system”) is shown in Fig. 5–9e. Here the contact force \( R \), which acts between \( A \) and \( B \), is considered as an internal force and hence is not shown on the free-body diagram. That is, it represents a pair of equal but opposite collinear forces which cancel each other.
EXAMPLE 5.4

Draw the free-body diagram of the unloaded platform that is suspended off the edge of the oil rig shown in Fig. 5–10(a). The platform has a mass of 200 kg.
EXAMPLE 5.4 CONTINUED

SOLUTION

The idealized model of the platform will be considered in two dimensions because by observation the loading and the dimensions are all symmetrical about a vertical plane passing through its center, Fig. 5–10b. The connection at A is considered to be a pin, and the cable supports the platform at B. The direction of the cable and average dimensions of the platform are listed, and the center of gravity G has been determined. It is from this model that we have drawn the free-body diagram shown in Fig. 5–10c. The platform’s weight is 200(9.81) = 1962 N. The force components $A_x$ and $A_y$ along with the cable force $T$ represent the reactions that both pins and both cables exert on the platform, Fig. 5–10a. Consequently, after the solution for these reactions, half their magnitude is developed at A and half is developed at B.
EXAMPLE 5.5

Determine the horizontal and vertical components of reaction on the beam caused by the pin at $B$ and the rocker at $A$ as shown in Fig. 5–12a. Neglect the weight of the beam.

**Fig. 5–12**

**SOLUTION**

**Free-Body Diagram.** Identify each of the forces shown on the free-body diagram of the beam, Fig. 5–12b. (See Example 5.1.) For simplicity, the 600-N force is represented by its $x$ and $y$ components as shown in Fig. 5–12b.
**Equations of Equilibrium.** Summing forces in the $x$ direction yields

$$\sum F_x = 0; \quad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \quad \text{Ans.}$$

A direct solution for $A_y$ can be obtained by applying the moment equation $\sum M_B = 0$ about point $B$.

$$\zeta + \sum M_B = 0; \quad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N} \quad \text{Ans.}$$

Summing forces in the $y$ direction, using this result, gives

$$\sum F_y = 0; \quad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \quad \text{Ans.}$$

**NOTE:** We can check this result by summing moments about point $A$.

$$\zeta + \sum M_A = 0; \quad -(600 \sin 45^\circ \text{ N})(2 \text{ m}) - (600 \cos 45^\circ \text{ N})(0.2 \text{ m})$$

$$-(100 \text{ N})(5 \text{ m}) - (200 \text{ N})(7 \text{ m}) + B_y(7 \text{ m}) = 0$$

$$B_y = 405 \text{ N} \quad \text{Ans.}$$
EXAMPLE 5.6

The cord shown in Fig. 5–13a supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.

![Diagram](image)

Fig. 5–13
EXAMPLE 5.6 CONTINUED

SOLUTION

**Free-Body Diagrams.** The free-body diagrams of the cord and pulley are shown in Fig. 5–13b. Note that the principle of action, equal but opposite reaction must be carefully observed when drawing each of these diagrams: the cord exerts an unknown load distribution $p$ on the pulley at the contact surface, whereas the pulley exerts an equal but opposite effect on the cord. For the solution, however, it is simpler to **combine** the free-body diagrams of the pulley and this portion of the cord, so that the distributed load becomes internal to this “system” and is therefore eliminated from the analysis, Fig. 5–13c.

**Equations of Equilibrium.** Summing moments about point $A$ to eliminate $A_x$ and $A_y$. Fig. 5–13c, we have

$\sum M_A = 0; \quad 100 \text{ lb} (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$

$T = 100 \text{ lb}$ \hspace{1cm} \text{Ans.}$

Using the result,

$\sum F_x = 0; \quad -A_x + 100 \sin 30^\circ \text{ lb} = 0$

$A_x = 50.0 \text{ lb}$ \hspace{1cm} \text{Ans.}$

$\sum F_y = 0; \quad A_y - 100 \text{ lb} - 100 \cos 30^\circ \text{ lb} = 0$

$A_y = 187 \text{ lb}$ \hspace{1cm} \text{Ans.}$

**NOTE:** It is seen that the tension remains constant as the cord passes over the pulley. (This of course is true for any angle $\theta$ at which the cord is directed and for any radius $r$ of the pulley.)
The member shown in Fig. 5–14a is pin-connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.
EXAMPLE 5.7 CONTINUED

SOLUTION

**Free-Body Diagram.** As shown in Fig. 5–14b, the reaction $N_B$ is perpendicular to the member at $B$. Also, horizontal and vertical components of reaction are represented at $A$.

**Equations of Equilibrium.** Summing moments about $A$, we obtain a direct solution for $N_B$,

$$\zeta + \sum M_A = 0; \quad -90 \text{ N m} - 60 \text{ N}(1 \text{ m}) + N_B(0.75 \text{ m}) = 0$$

$$N_B = 200 \text{ N}$$

Using this result,

$$\Rightarrow \sum F_x = 0; \quad A_x - 200 \sin 30^\circ \text{ N} = 0$$

$$A_x = 100 \text{ N} \quad \text{Ans.}$$

$$\uparrow \sum F_y = 0; \quad A_y - 200 \cos 30^\circ \text{ N} - 60 \text{ N} = 0$$

$$A_y = 233 \text{ N} \quad \text{Ans.}$$
EXAMPLE 5.8

The box wrench in Fig. 5–15a is used to tighten the bolt at A. If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to the bolt and the force of the wrench on the bolt.

SOLUTION

Free-Body Diagram. The free-body diagram for the wrench is shown in Fig. 5–15b. Since the bolt acts as a “fixed support,” it exerts force components $A_x$ and $A_y$ and a moment $M_A$ on the wrench at A.

Equations of Equilibrium.

\[ \pm \sum F_x = 0; \quad A_x - 52 \left( \frac{5}{13} \right) N + 30 \cos 60^\circ N = 0 \]

\[ A_x = 5.00 \text{ N} \quad \text{Ans.} \]

\[ + \sum F_y = 0; \quad A_y - 52 \left( \frac{12}{13} \right) N - 30 \sin 60^\circ N = 0 \]

\[ A_y = 74.0 \text{ N} \quad \text{Ans.} \]
\[ \zeta + \sum M_A = 0; \quad M_A = 52 \left( \frac{12}{13} \right) \text{N} (0.3 \text{ m}) - (30 \sin 60^\circ \text{ N})(0.7 \text{ m}) = 0 \]

\[ M_A = 32.6 \text{ N} \cdot \text{m} \quad \text{Ans.} \]

Note that \( M_A \) must be included in this moment summation. This couple moment is a free vector and represents the twisting resistance of the bolt on the wrench. By Newton’s third law, the wrench exerts an equal but opposite moment or torque on the bolt. Furthermore, the resultant force on the wrench is

\[ F_A = \sqrt{(5.00)^2 + (74.0)^2} = 74.1 \text{ N} \quad \text{Ans.} \]

**NOTE:** Although only three independent equilibrium equations can be written for a rigid body, it is a good practice to check the calculations using a fourth equilibrium equation. For example, the above computations may be verified in part by summing moments about point C:

\[ \zeta + \sum M_C = 0; \quad \left[ 52 \left( \frac{12}{13} \right) \text{N} \right] (0.4 \text{ m}) + 32.6 \text{ N} \cdot \text{m} - 74.0 \text{ N}(0.7 \text{ m}) = 0 \]

\[ 19.2 \text{ N} \cdot \text{m} + 32.6 \text{ N} \cdot \text{m} - 51.8 \text{ N} \cdot \text{m} = 0 \]
EXAMPLE 5.9

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in Fig. 5–16a.

SOLUTION

Free-Body Diagram. The free-body diagram is shown in Fig. 5–16b. The pin at A exerts two components of reaction on the member, $A_x$ and $A_y$.

Fig. 5–16
**Example 5.9 Continued**

**Equations of Equilibrium.** The reaction $N_B$ can be obtained *directly* by summing moments about point $A$ since $A_x$ and $A_y$ produce no moment about $A$.

\[
\zeta + \sum M_A = 0;
\]

\[
[N_B \cos 30^\circ](6 \text{ ft}) - [N_B \sin 30^\circ](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0
\]

\[
N_B = 536.2 \text{ lb} = 536 \text{ lb} \quad \text{Ans.}
\]

Using this result,

\[
\pm \sum F_x = 0; \quad A_x - (536.2 \text{ lb } \sin 30^\circ) = 0
\]

\[
A_x = 268 \text{ lb} \quad \text{Ans.}
\]

\[
\uparrow \sum F_y = 0; \quad A_y + (536.2 \text{ lb } \cos 30^\circ) - 750 \text{ lb} = 0
\]

\[
A_y = 286 \text{ lb} \quad \text{Ans.}
\]
EXAMPLE 5.10

The uniform smooth rod shown in Fig. 5–17a is subjected to a force and couple moment. If the rod is supported at $A$ by a smooth wall and at $B$ and $C$ either at the top or bottom by rollers, determine the reactions at these supports. Neglect the weight of the rod.

SOLUTION

Free-Body Diagram. As shown in Fig. 5–17b, all the support reactions act normal to the surfaces of contact since these surfaces are smooth. The reactions at $B$ and $C$ are shown acting in the positive $y'$ direction. This assumes that only the rollers located on the bottom of the rod are used for support.
**EXAMPLE 5.10 CONTINUED**

**Equations of Equilibrium.** Using the $x, y$ coordinate system in Fig. 5–17b, we have

1. $\sum F_x = 0; \quad C_y \sin 30^\circ + B_y \sin 30^\circ - A_x = 0$ \hspace{1cm} (1)
2. $\sum F_y = 0; \quad -300 \text{ N} + C_y \cos 30^\circ + B_y \cos 30^\circ = 0$ \hspace{1cm} (2)
3. $\sum M_A = 0; \quad -B_y (2 \text{ m}) + 4000 \text{ N} \cdot \text{m} - C_y (6 \text{ m})$
   \[ + (300 \cos 30^\circ \text{ N})(8 \text{ m}) = 0 \] \hspace{1cm} (3)

When writing the moment equation, it should be noted that the line of action of the force component $300 \sin 30^\circ \text{ N}$ passes through point $A$, and therefore this force is not included in the moment equation.

Solving Eqs. 2 and 3 simultaneously, we obtain

\[ B_y = -1000.0 \text{ N} = -1 \text{ kN} \] \hspace{1cm} Ans.

\[ C_y = 1346.4 \text{ N} = 1.35 \text{ kN} \] \hspace{1cm} Ans.

Since $B_y$ is a negative scalar, the sense of $B_y$ is opposite to that shown on the free-body diagram in Fig. 5–17b. Therefore, the top roller at $B$ serves as the support rather than the bottom one. Retaining the negative sign for $B_y$ (Why?) and substituting the results into Eq. 1, we obtain

\[ 1346.4 \sin 30^\circ \text{ N} + (-1000.0 \sin 30^\circ \text{ N}) - A_x = 0 \]

\[ A_x = 173 \text{ N} \] \hspace{1cm} Ans.
The uniform truck ramp shown in Fig. 5-18a has a weight of 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the tension in the cables.

**SOLUTION**

The idealized model of the ramp, which indicates all necessary dimensions and supports, is shown in Fig. 5-18b. Here the center of gravity is located at the midpoint since the ramp is considered to be uniform.

**Free-Body Diagram.** Working from the idealized model, the ramp’s free-body diagram is shown in Fig. 5-18c.

**Equations of Equilibrium.** Summing moments about point A will yield a direct solution for the cable tension. Using the principle of moments, there are several ways of determining the moment of T about A. If we use x and y components, with T applied at B, we have

\[
\zeta + \Sigma M_A = 0; \quad -T \cos 20^\circ (7 \sin 30^\circ \text{ ft}) + T \sin 20^\circ (7 \cos 30^\circ \text{ ft})
\]

\[
+ 400 \text{ lb} (5 \cos 30^\circ \text{ ft}) = 0
\]

\[
T = 1425 \text{ lb}
\]
EXAMPLE 5.11 CONTINUED

The simplest way to determine the moment of \( T \) about \( A \) is to resolve it into components along and perpendicular to the ramp at \( B \). Then the moment of the component along the ramp will be zero about \( A \), so that

\[
\zeta + \sum M_A = 0; \quad -T \sin 10^\circ (7 \text{ ft}) + 400 \text{ lb} (5 \cos 30^\circ \text{ ft}) = 0
\]

\[
T = 1425 \text{ lb}
\]

Since there are two cables supporting the ramp,

\[
T'' = \frac{T}{2} = 712 \text{ lb}
\]

\textit{Ans.}

\underline{NOTE}: As an exercise, show that \( A_x = 1339 \text{ lb} \) and \( A_y = 887.4 \text{ lb} \).
EXAMPLE 5.12

Determine the support reactions on the member in Fig. 5–19a. The collar at A is fixed to the member and can slide vertically along the vertical shaft.

Fig. 5–19

(a)

(b)

SOLUTION

Free-Body Diagram. The free-body diagram of the member is shown in Fig. 5–19b. The collar exerts a horizontal force $A_x$ and moment $M_A$ on the member. The reaction $N_B$ of the roller on the member is vertical.
EXAMPLE 5.12 CONTINUED

Equations of Equilibrium. The forces $A_x$ and $N_B$ can be determined directly from the force equations of equilibrium.

\[
\begin{align*}
\pm \sum F_x &= 0; \quad A_x = 0 \quad \text{Ans.} \\
\uparrow \downarrow \sum F_y &= 0; \quad N_B - 900 \text{ N} = 0 \\
& \quad N_B = 900 \text{ N} \quad \text{Ans.}
\end{align*}
\]

The moment $M_A$ can be determined by summing moments either about point $A$ or point $B$.

\[
\zeta + \sum M_A = 0;
\]

\[
M_A = 900 \text{ N}(1.5 \text{ m}) - 500 \text{ N} \cdot \text{m} + 900 \text{ N} [3 \text{ m} + (1 \text{ m}) \cos 45^\circ] = 0
\]

\[
M_A = -1486 \text{ N} \cdot \text{m} = 1.49 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\]

or

\[
\zeta + \sum M_B = 0; \quad M_A + 900 \text{ N} [1.5 \text{ m} + (1 \text{ m}) \cos 45^\circ] - 500 \text{ N} \cdot \text{m} = 0
\]

\[
M_A = -1486 \text{ N} \cdot \text{m} = 1.49 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\]

The negative sign indicates that $M_A$ has the opposite sense of rotation to that shown on the free-body diagram.