SYLLABUS

Instructors: Sedef Karakılıç and Engin Mermut  
Phones: (232) 412 85 91, 412 85 82  
e-mails: sedef.erim@deu.edu.tr and engin.mermut@deu.edu.tr  

Assistant: Didem Coşkun  
e-mail: didem.coskan@deu.edu.tr  
Phone: (232) 412 286 08

For announcements about the course, homeworks, results of your examinations, etc., see the COURSE WEB PAGE: [http://kisi.deu.edu.tr/engin.mermut/MAT2031-Analysis-I-2011g.html](http://kisi.deu.edu.tr/engin.mermut/MAT2031-Analysis-I-2011g.html)


- **GRADING:**

<table>
<thead>
<tr>
<th></th>
<th>Midterm</th>
<th>Final Examination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40%</td>
<td>60%</td>
</tr>
</tbody>
</table>

- **CONTENTS OF THE COURSE.**

The main topics that we shall study this term are the following for real-valued functions of a real variable: Sequences and series of real numbers; Continuous functions; Differentiation; Integration; Taylor series; Uniform convergence of sequences and series of functions.

In analysis, you will be asked to prove the results rigorously with every detail. We shall see rigorously all the topics that you have studied in your Calculus course. That is, the things you shall see will be familiar but now they will be studied more precisely with every detail. Of course, to do this part rigorously is harder than the Calculus course. It is of no surprise that students taking courses on analysis find it hard since the level of rigor is high and you are expected to understand the definitions, theorems, hypothesis, conclusions clearly to be able to use them precisely in your proofs. Do not get afraid. Indeed, the topics you shall see this term will seriously improve your mathematical reading and study; that is the way you must follow to obtain some mathematical maturity. As a mathematician, you must know how to state precisely and how to prove every result that you have seen in your Calculus course. For example, we have seen the motivation and idea for integration in your calculus course; we have seen the definition of the integrability of a function but we have not proved even the very often used properties of integration. In the calculus course, we were mostly interested in the computational parts; for example, to find integrals using the integration techniques were some of our main work but we have not proved even that a continuous function on a bounded closed interval is integrable. Or for example we have not proved the Extreme Value Theorem that says that a continuous real-valued function on a bounded closed interval attains its maximum and minimum values, but we were finding maximum and minimum values of differentiable functions using derivatives, critical points, etc. Of course, to prove all this, we must clearly start with the axioms for the real number system, and see how the properties of real numbers enable us to build the theory for limits, derivative and integral. The integration concept needs a finer limit idea that we are intuitively familiar from the Calculus course but now we shall work over the technical details. For the sequences and series of real numbers, and for power series, we have used some results in the Calculus course and we shall this term prove these results. We shall further study some topics that we have not seen in the Calculus course: uniform continuity of functions, uniform convergence of sequences and series of functions.

You must follow the textbooks weekly and study them always, working on the problems they contain to conceive the concepts of analysis and to develop your mathematical writing. On some Fridays, we shall have problem solving hours which you shall make practice of writing proofs in detail. There are exercises at the end of each section of each chapter of your textbooks. Do all of the exercises in your textbooks.

It is important that you do not miss the lectures and problem solving hours, otherwise it would be very hard for you to catch the lectures and to become successful. Attendance is important for you to catch the subjects we cover. Take these recommendations seriously.

→ See the next page for COURSE OUTLINE and some references.
References:


COURSE OUTLINE

- Axioms for the real number system. Completeness axiom, the least upper bound property; Existence of square roots; Natural numbers, induction; Archimedean property; Density of rational and irrational numbers in \( \mathbb{R} \); Inequalities and identities.
  \[ \rightarrow \text{Preliminaries, Appendix A and Chapter 1.} \]

- Sequences: Convergent sequences; Series; Sequential density of the rational numbers; The Monotone Convergence Theorem; The Nested Interval Theorem; The Sequential Compactness Theorem (Bolzano-Weierstrass Theorem); Compact subsets of \( \mathbb{R} \); Heine-Borel Theorem; Cauchy sequences, completeness of \( \mathbb{R} \); Cauchy convergence criterion for series; Basic Tests for convergence of series.
  \[ \rightarrow \text{Chapter 2 and Section 9.1.} \]

- Continuous Functions: Continuity; The Extreme Value Theorem; The Intermediate Value Theorem; Uniform Continuity; Sequential and \( \varepsilon-\delta \) definitions for continuity; Images and inverse images, monotone functions, continuity of inverse functions; Limits.
  \[ \rightarrow \text{Chapter 3.} \]

- Differentiation: Tangent lines and derivatives; Continuity of differentiable functions; Differentiating inverse functions and compositions (chain rule); The Mean Value Theorem and its geometric consequences; Strictly monotone functions; The Cauchy Mean Value Theorem and its analytic consequences; L’Hôpital’s Rule; The Leibnitz notation for derivatives.
  \[ \rightarrow \text{Chapter 4.} \]

- Transcendental Functions: The exponential and logarithm functions; Trigonometric functions: Inverse trigonometric Functions.
  \[ \rightarrow \text{Chapter 5.} \]

- Integration: Darboux sums, upper and lower integrals; The Archimedes-Riemann Theorem; Integrability of monotone functions; Leibnitz notation for integrals; Additivity, monotonicity and linearity of the integral; Continuity and integrability; The Fundamental Theorem of Calculus (integrating derivatives and differentiating integrals); The Mean Value Theorem for Integrals; Solution of differential equations; Integration by parts and by substitution; The number \( \pi \); The convergence of Darboux and Riemann Sums; The approximation of integrals (the Trapezoid Rule and Simpson’s Rule).
  \[ \rightarrow \text{Chapters 6 and 7.} \]

- Taylor Series: Approximation by Taylor polynomials; The Lagrange Remainder Theorem; Irrationality of the number \( e \); Euler’s constant \( \gamma \); The convergence of Taylor polynomials; The Cauchy Integral Remainder Theorem; Taylor series expansions of the elementary transcendental functions; Binomial series; Bernstein’s Theorem; A nonanalytic infinitely differentiable function; The Weierstrass Approximation Theorem.
  \[ \rightarrow \text{Chapter 8.} \]

- Series of Real Numbers: Series of nonnegative real numbers; Comparison and Limit Comparison Tests; Absolute and conditional convergence of series of real numbers; \( \limsup \) and \( \liminf \) of a sequence of real numbers; Extended root test; Ratio test; Basic and further tests for convergence of a series of real numbers; Rearrangement of a series; Riemann’s rearrangement theorem for conditionally convergent series; Alternating series; Abel’s partial summation formula; Dirichlet’s test; Estimation of series; Power series; The formula with \( \limsup \) for the radius of convergence of a power series; The interval of convergence of a power series; Stirling’s formula for \( n! \) and the gamma function \( \Gamma(x) \).
  \[ \rightarrow \text{Lecture Notes, and Section 2. 5 and Chapter 6 of the supplementary textbook by Wade.} \]

- Uniform Convergence: Sequences and series of functions; Pointwise and uniform convergence of sequences and series of functions; Uniform Cauchy criterion; Sufficient conditions for the continuity, integrability and differentiability of the uniform limit of functions; Weierstrass M-test for uniform convergence of a series of functions; Dirichlet’s Test for uniform convergence of a series of functions; Uniform convergence of power series; Term by term integration and differentiation of power series; Abel’s Theorem for the continuity of a power series on its interval of convergence; Product and division of power series; Analytic functions; A continuous but nowhere differentiable function.
  \[ \rightarrow \text{Chapter 9 of the textbook and Chapter 7 of the supplementary textbook by Wade.} \]

- Further topics if time permits: Improper integrals; Functions of bounded variation; Convex functions; Lebesgue’s criterion for integrability of a function on a bounded closed interval; Fourier Series; . . .