1. Consider the following function $f(x)$ defined by:

$$f(x) = \begin{cases} 
\frac{e^x - x - 1}{x^2}, & x \neq 0 \\
5, & x = 0.
\end{cases}$$

Is it continuous at $x = 0$? Find the required limit and explain your answer.
2. Suppose that for all $x$ in an open interval containing 1, the equation $y^2 + x^2e^y - 1 = 0$ has a unique solution for $y$ in $\mathbb{R}$ which we denote by $y = f(x)$. Further assume that $y = f(x)$ is a differentiable function of $x$. Thus we have a differentiable function $y = f(x)$ that has been implicitly defined for all $x$ in an open interval containing 1 such that

$$y^2 + x^2e^y - 1 = 0.$$

(a) Find $y' = \frac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find the equation of the tangent line at the point $(x_0, y_0) = (1, 0)$ on the curve defined by the above equation $y^2 + x^2e^y - 1 = 0$. 

3. Let \( g(x) = x^4 - 4x^3 + 4x^2 \), and \( F(x) = \int_0^{g(x)} e^t \, dt \) for all \( x \in \mathbb{R} \).

(a) \( F'(x) = ? \)

(b) On which intervals is \( F(x) \) increasing? Decreasing?

(c) At which points does \( F(x) \) have a local maximum or a local minimum value?
   Does \( F(x) \) have an absolute minimum value? Explain your answer.
4. Suppose that \( y = f(x) \) is a continuous function on the interval \([2, 5]\). Take the partition 
\( P = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6\} \) of the interval \([2, 5]\) where 
\[
x_k = 2 + k \frac{5 - 2}{6} = 2 + \frac{k}{2}, \quad k = 0, 1, 2, \ldots, 6.
\]
We are not given the formula for the function \( f(x) \) but we are given the following values of \( f \) at the following points:

<table>
<thead>
<tr>
<th>( x )</th>
<th>2.25</th>
<th>2.75</th>
<th>3.25</th>
<th>3.75</th>
<th>4.25</th>
<th>4.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>3</td>
<td>-4</td>
<td>-5</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

Approximate the value of the definite integral \( \int_2^5 f(x) \, dx \) by a Riemann sum for \( f \) on the interval \([2, 5]\) using these values of \( f \) and the partition \( P \).
5. Evaluate the following indefinite integrals:

(a) \[ \int \frac{\sqrt{x^4 - 1}}{x^4} \, dx = ? \text{ for } x \geq 1 \]  
(Hint: Use the substitution \( x = \sec \theta, 0 \leq \theta < \pi/2 \).)
(b) \[ \int \frac{4x^3 + 8x^2 + 8x + 20}{x^4 + 4x^2} \, dx = ? \]
6. Find the value of the improper integral \( \int_{1}^{\infty} \frac{\ln x}{x^2} \, dx \), if it is convergent.
7. Let $\mathcal{R}$ be the **region** enclosed by the parabola $x = y^2$ and the line $x - 2y = 3$. Let $\mathcal{M}$ be the **solid** obtained by revolving this region $\mathcal{R}$ around the $y$-axis.

(a) **Sketch** the region $\mathcal{R}$.
(b) Find the **area** of the region $\mathcal{R}$.
(c) **Sketch** the solid $\mathcal{M}$.
(d) Find the **volume** of the solid $\mathcal{M}$. 
8. Consider the epicycloid given by the following parametric equations

\[ x = 5 \cos t - \cos(5t), \quad y = 5 \sin t - \sin(5t), \quad t \in [0, \pi/2], \]

which draws the curve on the following figure.

Find the length of the epicycloid.
9. (a) **Sketch** the graph of the polar curve \( r = 1 + \sin \theta, \ 0 \leq \theta \leq 2\pi \).
(b) **Sketch** the region outside the polar curve \( r = 1 + \sin \theta \) and inside the polar curve \( r = 2 \), and find its **area**.