1. (a) State the \(\varepsilon-\delta\) definition of the limit of a function \(f\) at a point \(a\):
A function \(f(x)\) defined on an open interval about \(a\), except possibly at \(a\), is said to approach a real number \(L\) as \(x\) approaches \(a\) (written \(\lim_{x \to a} f(x) = L\)) if .................
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(b) State the definition of continuity of a function \(f\) at a point \(a\) in its domain:
A function \(f(x)\) is said to be \textit{continuous at a point} \(a\) in its domain if .................
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(c) For the following function $f(x)$, does there exist a real number $c$ such that $f(x)$ is continuous at 0:

$$f(x) = \begin{cases} 
(e^x + x)^{1/x}, & \text{if } x > 0 \\
 c, & \text{if } x = 0 \\
c \cos(x^{2/3} \ln |x|), & \text{if } x < 0 
\end{cases}$$
2. (a) Let \( a \in \mathbb{R} \) and \( f(x) \) be a function defined on an open interval containing \( a \). State the definition of differentiability of \( f(x) \) at \( a \). What is the definition of \( f'(a) \) in terms of a limit if \( f(x) \) is differentiable at \( a \)?

The function \( f(x) \) is said to be differentiable at \( a \) if .................................................................

If \( f(x) \) is differentiable at \( a \), then its derivative at \( a \) is defined to be

\[
f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}
\]

(b) Find \( f'(x) \) directly using the definition of the derivative in terms of limits for the function

\[ f(x) = x^5. \]
3. (a) Using the definition of the derivative, prove the Derivative Product Rule: If \( f(x) \) and \( g(x) \) are differentiable functions, then prove that \( f(x)g(x) \) is also a differentiable function and
\[
\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x).
\]

(b) Find the derivative of the function
\[
h(x) = \arctan(5x) \cdot \ln(\arcsin x) = \tan^{-1}(5x) \cdot \ln(\sin^{-1}(x)), \quad 0 < x < 1.
\]
by using the Derivative Product Rule and Chain Rule. You can use without proving the derivative formulae for \( \frac{d}{dx}(\tan^{-1} x), \frac{d}{dx}(\sin^{-1} x), \frac{d}{dx}(\ln x) \) and \( \frac{d}{dx}(ax^p) \) (for \( a > 0 \)).
4. Prove that the function

\[ f(x) = 2x - 1 + \arctan(x) = 2x - 1 + \tan^{-1}(x) \]

has exactly one real root, that is, there exists a unique real number \( x_0 \) such that \( f(x_0) = 0 \).
5. Suppose that for all $x$ in an open interval containing 1, the equation $y^3 + \cos(xy) - x^2 = 0$ has a unique solution for $y$ in $\mathbb{R}$ which we denote by $y = f(x)$. Further assume that $y = f(x)$ is a differentiable function of $x$. Thus we have a differentiable function $y = f(x)$ that has been implicitly defined for all $x$ in an open interval containing 1 such that

$$y^3 + \cos(xy) - x^2 = 0.$$

(a) Find $y' = \frac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find the equation of the tangent line at the point $(x_0, y_0) = (0, 1)$ on the curve defined by the equation $y^3 + \cos(xy) - x^2 = 0$. 
6. (a) Using the Chain Rule, find the derivative $f'(x)$ of the function

$$f(x) = \ln(\sec x + \tan x), \quad 0 < x < \pi/2.$$ 

You can of course use the derivative formulae for $\frac{d}{dx}(\ln x)$, $\frac{d}{dx}(\sec x)$ and $\frac{d}{dx}(\tan x)$.

(b) Is the function $f$ one-to-one on the interval $(0, \pi/2)$?

If so, find the derivative of its inverse function $f^{-1} : (0, \infty) \longrightarrow (0, \pi/2)$ using the rule for the derivative of the inverse function: $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$.

**Hint:** What is the sign of the derivative $f'(x)$ on the interval $(0, \pi/2)$? Is the function $f$ increasing/decreasing on $(0, \pi/2)$?
7. Find the height \( h \) and radius \( r \) of a right circular \textit{cylinder} of \textbf{MAXIMUM VOLUME} that can be inscribed in a sphere of radius \( \sqrt{3} \).
8. Draw the GRAPH of the function

\[ f(x) = 1 - 2e^x + xe^x \]

by following the graphing strategy.

**GRAPHING STRATEGY:**

(a) Identify the domain of \( f \) and any symmetries the curve may have.
(b) Find \( f' \) and \( f'' \).
(c) Find any critical points of \( f \) and identify the function’s behavior at each one.
(d) Find where the curve is increasing and where it is decreasing.
(e) Find the points of inflection, if any occur, and determine the concavity of the curve.
(f) Identify any asymptotes (horizontal asymptotes and vertical asymptotes).
(g) Plot key points, such as the intercepts and the points founds in the steps (c), (d) and (e), and plot the asymptotes if any.
(h) Gathering all you have found in the previous steps, sketch the curve.