Couples

Rotating forces: couples

(a)

(b)
Two parallel forces having an equal magnitude, opposite direction and not lying on the same line constitute a couple.

These two forces cannot be combined into a single force because their sum in every direction is zero.
The only effect of a couple on a rigid body is to produce a tendency of rotation or torsion; they have no force effect.

Because of this tendency to rotate, the couple is defined as a couple moment which exhibits this rotation.
The magnitude of the couple moment is equal to the magnitude of one of the forces times the perpendicular distance between the forces.

The sense of the moment is along the direction that is defined by the wrench rotating along the axis defined by the forces.
Let’s consider the action of two equal and opposite forces $\vec{F}$ and $-\vec{F}$ a distance $d$ apart. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the $\vec{M}$. This couple has a magnitude

$$M = F(a+d) - Fa = Fd$$
Note that the magnitude of the couple is independent of the distance $a$, which locates the forces with respect to the moment center $O$. The moment of a couple has the same value for all moment centers.

**While the moment of a force about a point is a sliding vector, couple moment is a free vector.**
In three dimensional applications, the couple moment is defined using vector algebra. Using the cross-product notation, the combined moment about point O of the forces forming the couple in the figure is

\[ \vec{M}_O = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = \vec{r}_A \times \vec{F} - \vec{r}_B \times \vec{F} = (\vec{r}_A - \vec{r}_B) \times \vec{F} \]

\[ \vec{r}_B + \vec{r} = \vec{r}_A \quad , \quad \vec{r}_A - \vec{r}_B = \vec{r} \]

\[ \vec{M}_O = \vec{r} \times \vec{F} \]
Couple is denoted in two and three dimensions as:

**Two dimensions**

- Counterclockwise couple
- Clockwise couple

**Three dimensions**
Couple moment is commonly indicated either as $\vec{M}$ or $\vec{C}$. Since couple moments are free vectors, it is possible to shift a couple moment or rotate it on the body it is applied, without altering the external effect of the couple on the body.
a) Original location

b) Couple can be translated to a parallel location in its plane or in a plane parallel to its original plane

c) Couple can be rotated within its plane

d) As long as the product $Fd$ stays unaltered, the magnitude of the couple force and the perpendicular distance between the forces can be varied
Addition of Couples

The couples \( \vec{C}_1, \vec{C}_2, \vec{C}_3, \ldots, \vec{C}_n \) acting on a body can be added to obtain the resultant couple \( \sum \vec{C} \). Since all of the couple components are free vectors, the resultant couple will also be a free vector.

\[
\sum \vec{C} = \sum \left( \vec{C}_1 + \vec{C}_2 + \vec{C}_3 + \ldots + \vec{C}_n \right)
\]

\[
\sum \vec{C} = \sum \vec{C}_x + \sum \vec{C}_y + \sum \vec{C}_z = \sum C_x \hat{i} + \sum C_y \hat{j} + \sum C_z \hat{k}
\]
1. As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force $F$ must be exerted by the ground on each of the main braked wheels at $A$ and $B$ to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel $C$, which is turned $90^\circ$ and unbraked.
2. During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially-inward component. Determine the moment exerted about the steering column at O.
3. Express and identify the resultant of the two forces and one couple shown acting on the shaft angled in the $x$-$z$ plane.
4. Determine the distance \( d \) between points \( A \) and \( B \) so that the resultant couple moment has a magnitude of \( M_R = 20 \text{ N} \cdot \text{m} \).
5. Three couples are formed by the three pairs of equal and opposite forces. Determine the resultant $M$ of the three couples.
6. If the magnitude of the moment of the tension force $\vec{T}$ acting at C about point B is 1150 N·m, determine $\vec{T}$ as a vector. Also determine the total moment of the couple acting on the surface and $\vec{T}$ about point B. Side AB of the surface lie in the $yz$ plane.