MOMENT
The assumption of particle acted on by concurrent forces is no longer valid for the rigid body which possesses a finite size and shape. In such systems $\sum \vec{F} = 0$ is a necessary but an insufficient condition. In addition to the tendency to move a body in the direction of its application, a second effect of a force which tends to rotate a body about an axis must also be considered. This rotational tendency is known as the \textit{moment}. 
The moment of a force about a point or an axis is the measure of that force to rotate the body around that point or axis.

*The axis may be any line which neither intersects nor is parallel to the line of application of the force.*

Moment is also referred to as “torque”.

Referring to the figure, the moment of force about point O is the measure of the force to rotate the body about axis AA, which passes through point O. Axis AA is perpendicular to the plane containing $\vec{F}$ and point O.
Moment possesses both magnitude and direction and is added according to the parallelogram law of vector addition. Therefore, moment is a \textit{vector quantity}. The magnitude of \( \vec{M} \) is defined as the product of the perpendicular distance \( d \) between point O, which is the intersection of axis AA with the plane and the line of action of \( \vec{F} \), times the magnitude of \( \vec{F} \).

\[
|\vec{M}| = M = Fd
\]

The unit of moment in SI system is N\cdot m.
**Point O** is defined as the “moment center”, distance *d* as the “moment arm”, and axis *AA* as the “moment axis”. The direction of *M* is determined by observing which direction force *F* is trying to rotate the body. When calculated in vector form, the direction of moment is determined by the right hand rule.

\[
\text{Plane containing point } O \text{ and force } F
\]

\[
|F| = F
\]
If forces acting in a single plane are considered, it is said that moment is taken about a point. In reality, however, moment is taken about an axis or a line perpendicular to the plane and which cuts the plane at that point.
Let’s consider the pipe wrench acted on by force $\vec{F}$ in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis OO perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm $d$, which is the perpendicular distance from the axis to the line of action of the force.
According to the right hand rule, for an object lying in the $xy$ plane, the force acting on the object will produce a moment perpendicular to the plane, that is in $z$ direction; the moment will be defined as

$$\vec{M} = M\hat{k} = (Fd)\hat{k}$$
In planar problems, only the magnitude of the moment may be taken into consideration. Its direction will be determined by the right hand rule. The direction may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments or vice versa. Sign consistency within a given problem is essential. The direction of the moment should be indicated either as clockwise (cw) or counterclockwise (ccw). A curved arrow is a convenient way to represent moments in two-dimensional analysis.
Vector Representation of Moment

In some two dimensional and many of the three dimensional problems, it is convenient to use a vector approach for moment calculations since it would be too difficult to determine the perpendicular distance $d$ between the moment center and the line of action of the force.

The moment of $\vec{F}$ about point O may be represented by the cross product expression:

$$\vec{M}_O = \vec{r} \times \vec{F}$$

where $\vec{r}$ is a position vector which runs from the moment center O to any point on the line of action of $\vec{F}$. 
Vector Representation of Moment

\[ \vec{M}_O = \vec{r} \times \vec{F} \]

where \( \vec{r} \) is a position vector which runs from the moment center \( O \) to any point on the line of action of \( \vec{F} \). Matrix representation for this product is

\[
\vec{M}_O = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
r_x & r_y & r_z \\
F_x & F_y & F_z
\end{vmatrix}
\]
The magnitude of the moment

\[ M_o = \left| \vec{r} \times \vec{F} \right| = \left| \vec{r} \right| \left| \vec{F} \right| \sin \alpha = Fd \]

It is important to recognize that the moment arm \( d=rsin\alpha \) does not depend on the particular point on the line of action of \( \vec{F} \) to which the vector \( \vec{r} \) is directed. The direction and sense of \( \vec{M} \) is established by applying the right hand rule to the sequence \( \vec{r} \times \vec{F} \). The moment vector will be perpendicular to the plane containing the vectors \( \vec{F} \) and \( \vec{r} \).
The sequence $\vec{r} \times \vec{F}$ must be maintained since $\vec{F} \times \vec{r}$ will produce a vector with a sense opposite to that of the correct moment.

$$-\vec{M}_o = \vec{F} \times \vec{r}$$

$\vec{r}$ is a position vector that starts from the moment center and ends at any point on the line of action of $\vec{F}$. Therefore, $\vec{r} = \vec{AB}$. Instead of the vector $\vec{AB}$, vectors $\vec{AC}$ or $\vec{AD}$ can also be employed as long as they are between the moment center the line of action of the force.
Varignon’s Theorem

The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point. In planar problems, if it is difficult to determine the perpendicular distance between the moment center and force, force can be written in terms of its components and the perpendicular distances between the components and the moment center can be considered.

\[
\vec{M}_o = \vec{r} \times \vec{R} \quad \vec{R} = \vec{P} + \vec{Q} \quad \vec{M}_o = \vec{r} \times (\vec{P} + \vec{Q}) = \vec{r} \times \vec{P} + \vec{r} \times \vec{Q}
\]

\[
M_o = R \cdot d = Q \cdot q - P \cdot p
\]
Varignon’s Theorem in Three Dimensions

Varignon’s theorem also applies in three dimensions: It states that the moment of the resultant force about a point is equal to the sum of the moments of its components about the same point or vice versa.

\[
\vec{M}_O = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \ldots + \vec{r} \times \vec{F}_n \\
= \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n) \\
\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \Sigma \vec{F} \\
\vec{M}_O = \vec{r} \times \Sigma \vec{F}
\]
Moment about an Axis or a Line in Space:

In order to determine the moment of $\vec{F}$ with respect to an axis $\lambda$ which is not perpendicular to the plane containing point O and force $\vec{F}$, first it is necessary to determine the moment about a point on that axis. Let’s assume this point is O.
Moment about an Axis or a Line in Space:

The moment taken about point O must be multiplied with the unit vector of axis \( \lambda \) using the dot product. The result of this dot product will give the magnitude of the projection of the moment which is parallel to the axis \( \lambda \). To determine the vector expression of this moment, the magnitude of the moment (projection of the moment about point O) must be multiplied with the unit vector of the axis \( \lambda \).

\[
\vec{M}_\lambda = \left( \vec{r} \times \vec{F} \cdot \vec{n}_\lambda \right) \vec{n}_\lambda
\]

where \( \vec{M}_O \) is the moment about point O.
EXAMPLES
1. Calculate the moment of the 250 N force on the handle of the monkey wrench about the center of the bolt.
2. Calculate the moment $M_o$ of the 250 N force about the base point O of the robot.
3. The spring-loaded follower $A$ bears against the circular portion of the cam until the lobe of the cam lifts the plunger. The force required to lift the plunger is proportional to its vertical movement $h$ from its lowest position. For design purposes determine the angle $\theta$ for which the moment of the contact force on the cam about the bearing $O$ is a maximum. In the enlarged view of the contact, neglect the small distance between the actual contact point $B$ and the end $C$ of the lobe.
4. The hydraulic cylinder $AB$ exerts a force $F$ of constant magnitude 2.5 kN directed from $A$ to $B$ as the engine is elevated. Determine the moment of $F$ about the point $C$ as a function of $\theta$ for the range $0 \leq \theta \leq 90^\circ$. What is the weight $W$ of the engine if the combined moment about $C$ of the force $F$ and weight $W$ for $\theta = 30^\circ$ is equal to zero?
5. The pipe assembly is subjected to an 80 N force. Determine the moment of this force about point A.
6. Strut AB of the 1 meter diameter hatch door exerts a force of 450 N on point B. Determine the moment of this force about point O. Line OB of the hatch door lies in the yz plane.
7. The access door is held in the $\alpha=36.87^\circ$ equilibrium position by chain AB. If the tension in the chain is $T=1430$ N, determine the scalar and vector expressions of the moment of this force about line OE. E is the midpoint of edge CD.
8. The arms $AB$ and $BC$ of a desk lamp lie in a vertical plane that forms an angle of $30^\circ$ with the $xy$ plane. To reposition the light, a force of magnitude $8$ N is applied as shown. Determine,

a) the moment of the force about point $A$,

b) the moment of the force about the axis of arm $AB$,

c) the angle between the line of action of force and line $AB$,

d) the perpendicular distance between the line $AB$ and line of action of force.

Direction $CD$ is parallel to the $z$-axis and lies in a parallel plane to the horizontal plane.

$AB = 450 \text{ mm}$, $BC = 325 \text{ mm}$
9. Concentrated force is acting perpendicular to the crank arm BC at point C. For the position $\theta=70^\circ$, what is $M_x$, the moment of about the x axis? At this instant, $M_y = -20 \text{ N} \cdot \text{m}$ and $M_z = -37.5 \text{ N} \cdot \text{m}$. 
10. Rectangular plate $ADCF$ is held in equilibrium by string $AB$ which has a tension of $T=14$ kN. Determine,

a) The magnitude of the moment of about the axis $DC$,

b) The perpendicular distance between lines $AB$ and $DC$. 
\[ \vec{T} = T\vec{n}_T = 14 \left[ \frac{6\vec{i} - 2\vec{j} - 3\vec{k}}{7} \right] \]
\[ \vec{T} = 12\vec{i} - 4\vec{j} - 6\vec{k} \quad \vec{n}_{DC} = \vec{n}_{EC} = \vec{n}_{AF} \]
\[ \vec{n}_{DC} = \left[ \frac{4\vec{i} + 4\vec{j} - 2\vec{k}}{6} \right] = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k} \]

\[
M_{DC} = \vec{M}_C \cdot \vec{n}_{DC} = (18\vec{i} + 12\vec{j} + 28\vec{k}) \cdot \left( \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k} \right)
\]
\[= 18 \left( \frac{2}{3} \right) + 12 \left( \frac{2}{3} \right) + 28 \left( -\frac{1}{3} \right)\]
\[M_{DC} = 12 + 8 - 9.33 = 10.67 \text{ kN} \cdot \text{m} \]
The parallel component of a force to a line does not generate moment with respect to that line. If $\vec{T}$ is resolved into two components parallel and normal to line DC, then

$$M_{DC} = T \sin \theta \cdot d$$

$$d = \frac{M_{DC}}{T \sin \theta} = \frac{10.67}{14 \sin 58.67} = 0.89 \text{ m}$$

The magnitude of moment $M_{DC}$ is equal to the normal component of $\vec{T}$ to line DC and the perpendicular distance between $\vec{T}$ and line DC.