THE DEVELOPMENT OF UTILITY THEORY. I

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But I have planted the tree of utility. I have planted it deep, and spread it wide.—BENTHAM.

The history of economic thought can be studied with many purposes. One may trace the effects of contemporary economic and social conditions on economic theory or—rather more bravely—the effects of economic theories on economic and social developments. One may study the history to find the original discoverers of theories, spurred on by the dream of new Cantillons; or one may compare the economics of the great economists with that of the rank and file, as a contribution to the structure and process of intellectual change. Or one may, and most often does, simply set forth the major steps in the development of a branch of economic theory, hoping that it can be justified by its contribution to the understanding of modern economics. This history of utility theory is offered primarily with this last purpose, although in the final section I review the history to answer the question, “Why do economists change their theories?”

The scope of this study is limited in several respects. First, it covers primarily the period from Smith to Slutsky, that is, from 1776 to 1915. Second, the study is limited to certain important topics and to the treatment of these topics by economists of the first rank. The application of utility theory to welfare economics is the most important topic omitted. An estimate of the part played by utility theory in forming economists’ views of desirable social policy is too large a task, in the complexity of issues and volume of literature involved, to be treated incidentally. The omission is justified by the fact that most economists of the period used utility theory primarily to explain economic behavior (particularly demand behavior) and only secondarily (when at all) to amend or justify economic policy.1

1. THE CLASSICAL BACKGROUND

ADAM SMITH

Drawing upon a long line of predecessors, Smith gave to his immediate successors, and they uncritically accepted, the distinction between value in use and value in exchange:

The word value, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called “value in use”; the other, “value in exchange.” The things which have the greatest value in use have frequently little or no value in exchange; and on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on

1I have also omitted consideration of the criticisms raised by the antitheoretical writers, who played no constructive part in the development of the theory. For a discussion of some of their views see J. Viner, “The Utility Theory and Its Critics,” Journal of Political Economy, XXXIII (1925), 369–87.

I wish to acknowledge the helpful suggestions of Arthur F. Burns, Milton Friedman, and Paul A. Samuelson.
the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it. 3

The fame of this passage rivals its ambiguity.

The paradox—that value in exchange may exceed or fall short of value in use—was, strictly speaking, a meaningless statement, for Smith had no basis (i.e., no concept of marginal utility of income or marginal price of utility) on which he could compare such heterogeneous quantities. On any reasonable interpretation, moreover, Smith's statement that value in use could be less than value in exchange was clearly a moral judgment, not shared by the possessors of diamonds. To avoid the incomparability of money and utility, one may interpret Smith to mean that the ratio of values of two commodities is not equal to the ratio of their total utilities. 3 On such a reading, Smith's statement deserves neither criticism nor quotation.


3 Or, alternatively, that the ratio of the prices of two commodities is not equal to the ratio of their total utilities: but this also requires an illegitimate selection of units: The price of what quantity of diamonds is to be compared with the price of one gallon of water? Smith makes such illegitimate statements; for example, "The whole quantity of a cheap commodity brought to market, is commonly not only greater, but of greater value, than the whole quantity of a dear one. The whole quantity of bread annually brought to market, is not only greater, but of greater value than the whole quantity of butcher's-meat; the whole quantity of butcher's meat, than the whole quantity of poultry; and the whole quantity of poultry, than the whole quantity of wild fowl. There are so many more purchases for the cheap than for the dear commodity, that, not only a greater quantity of it, but a greater value, can commonly be disposed of" (Ibid., p. 212; see also p. 238).

Nevertheless, this statement can be reformulated into a meaningful and interesting hypothesis: Order commodities by the income class of consumers, using the proportion of families in the income class that purchase the commodity as the basis for choosing the income class. Then does aggregate value of output fall as income class rises?

This passage is not Smith's title to recognition in our history of utility. His role is different: it is to show that demand functions, as a set of empirical relationships, were already an established part of economic analysis. The negatively sloping demand curve was already axiomatic; for example, "A competition will immediately begin among [the buyers when an abnormally small supply is available], and the market price will rise more or less above the natural price." 4 The effect of income on consumption was not ignored:

The proportion of the expense of house-rent to the whole expense of living, is different in the different degrees of fortune. It is perhaps highest in the highest degree, and it diminishes gradually through the inferior degrees, so as in general to be lowest in the lowest degree. The necessaries of life occasion the great expense of the poor. They find it difficult to get food, and the greater part of their little revenue is spent in getting it. The luxuries and vanities of life occasion the principal expense of the rich; and a magnificent house embellishes and sets off to the best advantage all the other luxuries and vanities which they possess. A tax upon house-rents, therefore, would in general fall heaviest upon the rich; and in this sort of inequality there would not, perhaps, be any thing very unreasonable. 5

This type of demand analysis was continued and improved by Smith's successors, but his example should suffice to remind us that a history of utility is not a history of demand theory.

BENTHAM

Jeremy Bentham brought the principle of utility (to be understood much more broadly than is customary in economics) to the forefront of discussion in

4 Ibid., p. 56. Substitution is illustrated by the effects of a royal death on the prices of black and colored cloth (Ibid., p. 50).

5 Ibid., pp. 73-94. This is of course the opposite of modern budgetary findings, but near-contemporary budget studies seem to me indirectly to support Smith.
England at the beginning of the nineteenth century. In his *Introduction to the Principles of Morals and Legislation* (1789) he suggested the measurement of quantities of pleasure and pain (primarily for the purpose of constructing a more rational system of civil and criminal law). Four dimensions of pleasure and pain were distinguished for the individual: (1) intensity, (2) duration, (3) certainty, and (4) propinquity.

The first two dimensions are clearly relevant to the measurement of a pleasure, but the latter two are better treated as two of the factors which influence an individual’s response to a particular pleasure or pain. Bentham did not give explicit directions for calculating a given pleasure and indeed devoted a long chapter (vi) to “Circumstances Influencing Sensibility,” which listed no less than thirty-two circumstances (such as age, sex, education, and firmness of mind) that must be taken into account in carrying out such a calculation.

The theory was much elaborated with respect to economic applications in *Traité de législation* (1832); a lucid synthesis of many manuscripts made by his disciple, Étienne Dumont. Bentham was particularly concerned with the problem of equality of income, and this

1. *Op. cit.,* ch. iv. In addition, four further “dimensions” were added for the appraisal of the total satisfaction of an “act”: the consumption of a loaf of bread might be the pleasure to which the first four dimensions refer; the theft of the loaf might be the act. These additional dimensions were secundity and purity; respectively, the chance of one pleasure leading to another and the chance of a pleasure not being followed by a pain.

2. As Bentham indicated elsewhere (see *Works of Jeremy Bentham* [Edinburgh: Tait, 1843], I, 205; III, 322).


raised the question of comparisons of the utilities of persons who might differ in thirty-two circumstances:

It is to be observed in general, that in speaking of the effect of a portion of wealth upon happiness, abstraction is always to be made of the particular sensibility of individuals, and of the exterior circumstances in which they may be placed. Differences of character are inscrutable; and such is the diversity of circumstances, that they are never the same for two individuals. Unless we begin by dropping these two considerations, it will be impossible to announce any general proposition. But though each of these propositions may prove false or inexact in a given individual case, that will furnish no argument against their speculative truth and practical utility. It is enough for the justification of these propositions—1st, If they approach nearer the truth than any others which can be substituted for them; and, 2nd, with less inconvenience than any others they can be made the basis of legislation.

Thus, he achieved interpersonal comparisons, not by calculation, but by assumption, justified by the desirability (somehow determined) of its corollaries. This resort to a question-begging assumption was a fundamental failure of his project to provide a scientific basis for social policy: the scientific basis was being justified by the policies to which it led. In one of his manuscripts he argued that this assumption was merely an abbreviation and that the conclusions he deduced could be reached (more laboriously) without it, which is not in general true.


5. “It is in vain to talk of adding quantities which after the addition will continue distinct as they were before, one man’s happiness will never be another man’s happiness; a gain to one man is no gain to another; you might as well pretend to add 20 apples to 10 pears, which after you had done that could not be 40 of any one thing but 20 of each just as there was before. This addibility of the happiness of different subjects, however, when considered rigorously it may appear fictitious, is a postulateum without the allowance of which all political reasoning is at a stand; nor is it more
Having surmounted this obstacle no better than subsequent economists, Bentham proceeded to establish a set of propositions on the utility of income: 11

1st. Each portion of wealth has a corresponding portion of happiness.

2nd. Of two individuals with unequal fortunes, he who has the most wealth has the most happiness.

3rd. The excess in happiness of the richer will not be so great as the excess of his wealth. 12

Each of these propositions was elaborated, and the utility calculus was used to defend equality (“The nearer the actual proportion approaches to equality, the greater will be the total mass of happiness”)2, although equality was finally rejected in favor of security of property. As corollaries, gambling was utility-decreasing and insurance utility-increasing. 13

In a manuscript written about 1782, Bentham attempted to set forth more clearly the precise measurement of utility. 14 We are given a definition of the unit of intensity:

The degree of intensity possessed by that pleasure which is the faintest of any that can be distinguished to be pleasure, may be represented by unity. Such a degree of intensity is in every day’s experience: according as any pleasures are perceived to be more and more intense, they may be represented by higher and higher numbers; but there is no fixing upon any particular degree of intensity as being the highest of which a pleasure is susceptible. 15

(This suggested measure will be discussed in connection with the Weber-Fechner literature.) Then, shifting ground, Bentham argues that, although utility does not increase as fast as income, for small changes the two move proportionately, 16 so we may measure pleasures through the prices they command:

If then between two pleasures the one produced by the possession of money, the other not, a man had as lief enjoy the one as the other, such pleasures are to be reputed equal. But the pleasure produced by the possession of money, is as the quantity of money that produces it; money is therefore the measure of this pleasure. But the other pleasure is equal to this; the other pleasure therefore is as the money that produces this; therefore money is also the measure of that other pleasure. 17

Unfortunately, this procedure is illegitimate; we cannot use an equality (or, more strictly, a constancy of the marginal utility of money) that holds for small changes to measure total utilities. 18 These suggestions are import-

11 Theory of Legislation, pp. 103 ff.; all statements italicized by Bentham.

12 The use of marginal analysis was even more explicit in his Pammeligrammic Fragments:

But the quantity of happiness will not go on increasing in anything near the same proportion as the quantity of wealth—ten thousand times the quantity of wealth will not bring with it ten thousand times the quantity of happiness. It will even be matter of doubt whether ten thousand times the wealth will in general bring with it twice the happiness.

13 "... the quantity of happiness produced by a particle of wealth (each particle being of the same magnitude) will be less and less at every particle;..." (Works, III, 229; see also IV, 541).


15 Ibid., p. 392.

16 Ibid., p. 408.

17 Ibid., p. 410.

18 Bentham appears to have recognized this difficulty when, in a passage following a discussion of
tant chiefly in revealing Bentham's awareness of the crucial problems in his calculus and his ingenuity in attempting to solve them.18

Bentham had indeed planted the tree of utility. No reader could overlook the concept of utility as a numerical magnitude; and the implications for economic analysis were not obscure. But they were overlooked.

THE RICARDIANS

The economists of Bentham’s time did not follow the approach he had opened. One may conjecture that this failure is due to the fact that Ricardo, who gave the economics of this period much of its slant and direction, was not a Benthamite. It is true that he was the friend of Bentham and the close friend of James Mill, Bentham’s leading disciple. Yet there is no evidence that he was a devout utilitarian and much evidence that he was unphilosophical—essentially a pragmatic reformer.20

It is clear, in any event, that Ricardo did not apply the utility calculus to economics. He began his Principles with the quotation of Smith’s distinction between value in use and value in exchange and ended the volume with the statement: “Value in use cannot be measured by any known standard; it is
differently estimated by different persons.”21 I should be content to notice that he left the theory of utility as highly developed as he found it—as much cannot be said for the theory of value—were it not for a remarkable interpretation of Marshall’s:

Again, in a profound, though very incomplete, discussion of the difference between “Value and Riches” he seems to be feeling his way towards the distinction between marginal and total utility. For by Riches he means total utility, and he seems to be always on the point of stating that value corresponds to the increment of riches which results from that part of the commodity which it is only just worth the while of purchasers to buy; and that when the supply runs short, whether temporarily in consequence of a passing accident, or permanently in consequence of an increase in cost of production, there is a rise in that marginal increment of riches which is measured by value, at the same time that there is a diminution in the aggregate riches, the total utility, derived from the commodity. Throughout the whole discussion he is trying to say, though (being ignorant of the terse language of the differential calculus) he did not get hold of the right words in which to say it neatly, that marginal utility is raised and total utility is lessened by any check to supply.22

In the chapter (xx) referred to, Ricardo defines riches as “necessaries, conveniences, and amusements,” and value, as usual, is measured by the amount of labor necessary to produce a commodity. The chapter is essentially an exercise in the paradoxes of this definition of value; for example, if the productivity of labor doubles, riches double, but value changes only if the number of laborers changes. We may properly identify “necessaries, conveniences, and amusements” with total


utility; but what of marginal utility? Ricardo says that, if a person receives two sacks of corn where formerly he received one, "he gets, indeed double the quantity of riches—double the quantity of utility—double the quantity of what Adam Smith calls value in use." Hence he did not believe that marginal utility diminishes as quantity increases.

He continued:

When I give 2,000 times more cloth for a pound of gold than I give for a pound of iron, does it prove that I attach 2,000 times more utility to gold than I do to iron? certainly not; it proves only as admitted by M. Say, that the cost of production of gold is 2,000 times greater than the cost of production of iron . . . if utility were the measure of value, it is probable I should give more for the iron.\footnote{Principles, p. 265.}

The writer of this passage cannot be said to have been close to the notion of marginal utility. I cannot find a single sentence that gives support to Marshall's interpretation, and I think that it should be added to the list of examples of his peculiar documentation and interpretation of predecessors.

Ricardo's influence was such that James Mill, the logical person to apply Bentham's system to economics, was content to present a rigid simplification of Ricardo's Principles,\footnote{Ibid., pp. 267–68.} and his son—whose formative work in economics, we must remember, came chiefly in the 1830's—did little more with utility.\footnote{In his Elements of Political Economy (3d ed., 1827).} Only the French utilitarian, J. B. Say, attempted to give utility a substantial place in economic theory, and he was prevented from doing so effectively by his inability to arrive at a notion of marginal analysis. In order to support the thesis that prices are proportional to utilities, he was driven to invent the metaphysical distinction between natural and social wealth:

One pays 2,000 times as much for a pound of gold as for a pound of iron. Here is how, on my theory, this phenomenon is explained. I assume with you that a pound of iron has the same utility as a pound of gold, although it is worth only one-two-thousandth as much. I say that there are in the iron 1,999 degrees of utility that nature has given us without charge, and 1 degree that we create by work, at an expense that we will assume only if a consumer is willing to reimburse us; hence the pound of iron has 2,000 degrees of utility. The gold also has 2,000 degrees of utility (on your assumption), which however can be obtained only on exacting terms, that is to say, . . . by expenses of 2,000. The 1,999 degrees of utility for which we do not pay when we consume iron are part of our natural wealth. . . . The single degree of utility which must be paid for is part of our social wealth.\footnote{Principles of Political Economy (Ashley ed.; New York: Longmans, Green, 1929), pp. 442–44, 804.}

II. THE UNSUCCESSFUL DISCOVERERS

The principle that equal increments of utility-producing means (such as income or bread) yield diminishing increments of utility is a commonplace. The first statement in print of a commonplace is adventitious; it is of no importance in the development of economics, and it confers no intellectual stature on its author. The statement acquires interest only when it is logically developed or explicitly applied to economic problems, and it acquires importance only when a considerable number of economists are persuaded to incorporate it into their analyses. Interest and importance are of course matters of degree.

Some economists gave clear state-
ments of the principle of diminishing marginal utility but did not apply it to economic problems; they include Lloyd (1833), Senior (1836), Jennings (1855), and Hearn (1864). Others applied utility theory to economic events without explicitly developing the principle of diminishing marginal utility: A. Walras (1831) and Longfield (1834), for example. At least two economists—in addition to Bentham—elaborated the principle or applied it to economic problems but failed to persuade other economists of its usefulness. Their theories will be summarized briefly.

Jules Dupuit, a distinguished engineer, was led to the marginal utility theory by his attempt to construct a theory of prices that maximize utility. He distinguished total and marginal utility with great clarity and discovered “une espèce de bénéfice” that we now call consumers' surplus. It was defined as the excess of total utility over marginal utility times the number of units of the commodity, but it was actually taken to be the area under the demand curve minus the expenditures on the commodity (i.e., Marshall's measure without his restrictions).

Armed with this concept, he investigated the optimum toll on a bridge.

![Diagram](image)

His analysis was as follows. Let $NP$ be the demand (and marginal utility) curve, $Op$ the price (Fig. 1). Then $OmnP$ is the absolute utility consumers obtain from the use of the bridge, and $pnP$ is the relative utility. If the toll is reduced by $pp'$, there is a net gain of consumer utility of $qmn'$ (equal to the area under the demand curve between $r$ and $r'$ minus the expenditure $rr'nq'$).

Dupuit's general conclusion is: “The utility of a means of communication, and in general of any product, is at a

Dupuit's instruction for measuring utility reveals the tacit identification of utility and demand curves: “Assume that all the like commodities whose general utility one wishes to determine are subjected to a tax which is increased by small steps. At each increase, a certain quantity of the commodity will no longer be purchased. The utility of this quantity in terms of money will be the quantity multiplied by the tax. By increasing the tax until all purchases cease, and adding the partial products, one will obtain the total utility of the commodity” (ibid., p. 50; also p. 180).
maximum when the toll or the price is zero." This is little more than a tautology, and Dupuit did not draw the further and illegitimate conclusion that the optimum toll rate is zero:

It will not be our conclusion [that tolls should be small or zero], when we treat of tariffs, but we hope to have demonstrated that [tariff rates] must be studied, combined on rational principles to produce simultaneously the greatest possible utility and a revenue which will repay the expense of maintenance and the interest on the capital investment.\(^{34}\)

We see that he was not afraid of interpersonal comparisons of utility, and in fact he argued that the effects of price changes on the distribution of income must be ignored because they were merely transfers.\(^{35}\)

Dupuit could not reach a complete theory of optimum prices because he did not devise a coherent theory of cost.\(^{36}\) One is impressed by the narrowness of

\(^{31}\) Ibid., p. 161. I have transposed the axes of Dupuit’s diagram.

\(^{32}\) Ibid., p. 57. Elsewhere he says that the ideal toll would be one proportional to the consumers’ total utility, but this is impracticable because of “l’impossibilité universelle” (ibid., p. 144); and the effects of alternative methods of financing public works (e.g., the incidence of taxes) must be studied before a practical recommendation can be made (ibid., p. 161). Multiple price systems were also considered (ibid., pp. 64–65, 140 ff.).

\(^{33}\) Ibid., p. 57.

\(^{34}\) This is illustrated by the following quotation, in which price fluctuations are treated as exercises of arbitrary power:

“In order that there be an increase or decrease in utility, it is necessary that there be a decrease or increase in [a commodity’s] cost of production—there being no change in its quality. When there are only variations in market price [prix vénal], the consumer gains what the producer loses, or conversely. Thus, when an article costing 20 francs to produce is sold for 50 francs, as a result of a monopoly or concession, the producer deprives every buyer of 30 francs of utility. If some circumstance forces him to lower his price by 10 francs, his income diminishes by 10 francs per unit and that of each buyer increases by 10 francs. There is a cancellation; no utility is produced” (ibid., pp. 52–53).

His vision; the explicit formulation of the concept of consumer surplus is elegant, but there is no intuition of the difficulties in the concept, nor is there an attempt to construct the larger theoretical framework necessary to solve his problem.

**Gossen (1854)**

Heinrich Gossen is one of the most tragic figures in the history of economics. He was a profound, original, and untrained thinker who hid his thoughts behind painfully complex arithmetical and algebraic exercises.\(^{37}\) He displayed every trait of the crank,\(^{38}\) excepting only one: history has so far believed that he was right. Only a few distinctive features of his work will be commented upon.

First, Gossen’s discussion of the laws of satisfaction is concerned only with individual acts of consumption, such as the eating of slices of bread.\(^{39}\) Correspondingly, in his early diagrams marginal utility is a function of time (duration of the act of consumption), and only after a considerable elaboration of this approach does he take quantity of a (perishable) commodity as proportional to duration of consumption.\(^{40}\)

\(^{35}\) Only a person who has labored through the volume can savor the magnificent understatement of Edgeworth: “He may seem somewhat deficient in the quality of mathematical elegance” (“Gossen,” Palgrave’s Dictionary of Political Economy [London: Macmillan, 1923], II, 232).

\(^{36}\) *Hs Entwickelung der Gesetze des menschlichen Verkehrs* (3d ed.; Berlin: Prager, 1937), which is not encumbered with chapters, begins with the famous sentences: “On the following pages I submit to public judgment the result of 20 years of meditation. What a Copernicus succeeded in explaining of the relationships of worlds in space, that I believe I have performed for the explanation of the relationships of men on earth.”


\(^{38}\) *Entwickelung*, p. 29; his treatment of durable goods is not sound (see pp. 25, 29–30).
Yet he does not attempt to work out a theory of the temporal pattern of consumption, and this portion of his theory seems misdirected.

Second, he presents a theory of the marginal disutility of labor that is completely symmetrical with that of the marginal utility of consumer goods. Gossen's curve of the marginal disutility of income is essentially identical with that which Jevons made famous: the early hours of work yield utility, but as the duration of labor increases, the marginal utility diminishes to zero and then to negative values. He defines the condition of maximum utility as that in which the marginal utility of a unit of product is numerically equal to the marginal disutility of the labor necessary to produce a unit of product.

Third, Gossen was the first writer to formulate explicitly what I shall call the fundamental principle of marginal utility theory:

A person maximizes his utility when he distributes his available money among the various goods so that he obtains the same amount of satisfaction from the last unit of money (Geldatom) spent upon each commodity.

We may translate this statement into semisymbolic form:

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \frac{MU_3}{p_3} = \ldots,$$

where $MU_i$ represents the marginal utility of the $i$th commodity and $p_i$ its price. (We shall adhere to the notation: $x_i$ is the quantity of commodity $X_i$, $p_i$ is its price, $MU_i$ is its marginal utility, and $R$ is money income.) This equation marked a long step forward in the development of the relationship between utility and demand curves.

Finally, Gossen's views on the measurability of utility are vague but tantalizing:

We can conceive of the magnitudes of various pleasures only by comparing them with one another, as, indeed, we must also do in measuring other objects. We can measure the magnitudes of various areas only by taking a particular area as the unit of measurement, or the weights of different bodies only by taking a particular weight as the unit. Similarly, we must fix on one pleasure as our unit, and hence an indefiniteness remains in the measurement of a pleasure. It is a matter of indifference which pleasure we choose as the unit. Perhaps the consequences will be most convenient if we choose the pleasure from the commodity which we use as money.

He did not notice that there might be no unit of utility comparable with that of area or weight; and it is probably going too far to read into this passage the later position that it is sufficient to deal with the ratios of marginal utilities.

III. THE BEGINNINGS OF THE MODERN THEORY

The utility theory finally began to win a place in generally accepted economics in the 1870's, under the triple auspices of Jevons, Menger, and Walras. Independently these economists arrived at positions similar in the main and sometimes in detail. I shall compare their treatments of certain basic

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41 Ibid., p. 36.
42 Ibid., p. 45.
43 Ibid., pp. 91–94.
44 Ibid., p. 123.
45 Marshall was a contemporary discoverer of the theory but did not publish it until later (Memorials of Alfred Marshall [London: Macmillan, 1925], p. 22). J. B. Clark was a somewhat later discoverer and never developed the theory to a level comparable with the best contemporary European analysis. He became preoccupied with a neglected problem to which he could not find a useful solution: how to apply marginal analysis to variations in the quality of goods (see The Philosophy of Wealth [Boston: Ginn & Co., 1892], Preface and p. 76 n.; Distribution of Wealth [New York: Macmillan, 1931], chaps. xiv–xvi).
problems of the theory, and henceforth our organization will be by subject.

A. CRITICISM OF RECEIVED DOCTRINE

Each of these founders of utility theory criticized the Ricardian theory of value, but for each this was an incidental and minor point; they deemed the positive merits of the utility theory a sufficient basis for acceptance. Thus, only after completing the presentation of his utility theory did Jevons point out the deficiencies in Ricardo’s labor value theory. These deficiencies were three: (1) Ricardo required a special theory for commodities with fixed supplies, such as rare statues. This proved that labor cost is not essential to value. (2) Large labor costs will not confer high value on a commodity if the future demand is erroneously forecast; “in commerce bygones are for ever bygones.” (3) Labor is heterogeneous, and the various types of labor can be compared only through the values of their products. On the other hand, the cost of production theory of value fits in nicely as a special case of the utility theory, for it explains the relative quantities of commodities that will be supplied.

Menger and Walras took fundamentally the same position. The former also gave the first two criticisms listed above and, in addition, made a parallel criticism to the Ricardian rent theory: if the value of land did not depend upon labor cost, this demonstrated a serious lack of generality in the classical theory of value. Walras repeated the criticism that the classical theory lacked generality, emphasized the reciprocal effects of prices of products and of productive services on one another, and denied the existence of the class of commodities whose supplies could be infinitely increased, on the overly literal ground that no productive resource was available in infinite quantity.

The task of elaborating and expounding the theory, and of exaggerating its merits and understating the usefulness of the classical theory—the inevitable accompaniments of intellectual innovations—fell largely to disciples, in particular Wieser and Böhm-Bawerk. These men did not improve on the substance of the theory—in fact, it deteriorated in their hands—so we shall pass them by.

II. THE EXISTENCE AND MEASURABILITY OF UTILITY

Without exception, the founders accepted the existence of utility as a fact of common experience, congruent with the most casual introspection. Jevons was most explicit:

The science of Economics, however, is in some degree peculiar, owing to the fact that its ultimate laws are known to us immediately by intuition, or, at any rate, they are furnished to us ready made by other mental or physical sciences.

... The theory here given may be described as the mechanics of utility and self-interest. Oversights may have been committed in tracing

47 Ibid., p. 166.
48 Ibid., p. 165.
49 Grundzüge der Volkswirtschaftslehre (Vienna: Braumüller, 1871), pp. 69, 130-21, 144-45.
out its details, but in its main features this theory must be the true one. Its method is as sure and demonstrative as that of kinematics or statics, nay, almost as self-evident as are the elements of Euclid. . . .\(^52\)

I am inclined to interpret the silence of Menger and Walras on the existence of utility as indicative of an equally complete acceptance.

Menger glossed over the problem of measurability of utility. He represented marginal utilities by numbers and employed an equality of marginal utilities in various uses as the criterion of the optimum allocation of a good.\(^53\) His word for utility — *Bedeutung* — was surely intentionally neutral, but probably it was chosen for its nonethical flavor.\(^54\) Walras was equally vague; he simply assumed the existence of a unit of measure of intensity of utility and thereafter spoke of utility as an absolute magnitude.\(^55\)

Jevons’ attack on the problem of measurability was characteristically frank and confused. He denied that utility was measurable:

There is no unit of labour, or suffering, or enjoyment.

I have granted that we can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of question.\(^56\)

Yet he seemed also to argue that one cannot be sure that utility is not measurable but only that it could not presently be measured.\(^57\) He was somewhat more skeptical of the measurability of utility in the first (1871) than in the second (1879) edition; for example, in the second edition he deleted the following passage:

I confess that it seems to me difficult even to imagine how such estimations [of utility] and summations can be made with any approach to accuracy. Greatly though I admire the clear and precise notions of Bentham, I know not where his numerical data are to be found.\(^58\)

With gallant inconsistency, he proceeded to devise a way to measure utility. It employed the familiar measuring rod of money:

It is from the quantitative effects of the feelings that we must estimate their comparative amounts.

I never attempt to estimate the whole pleasure gained by purchasing a commodity; the theory merely expressed that, when a man has purchased enough, he would derive equal pleasure from the possession of a small quantity more as he would from the money price of it.\(^59\)

This position is elaborated ingeniously: We can construct a demand curve by observation (or possibly experiment), and then we can pass to the marginal utility curve by means of the equation,

\[ MU' = MU, \]

where \( MU' \) is the marginal utility of income.\(^60\)

For the first approximation we may assume that the general utility of a person's income is not affected by the changes of price of the commodity. . . .

The method of determining the function of utility explained above will hardly apply, however, to the main elements of expenditure. The price of bread, for instance, cannot be properly brought under the equation in question, because, when the price of bread rises much, the resources of poor persons are strained, money


\(^{54}\) On one occasion he states that his numbers represent only relative utilities and that numbers such as 80 and 40 indicate only that the former (marginal) utility is twice as large as the latter (ibid., p. 163 n.).

\(^{55}\) *Elements*, pp. 74, 103, 159.


\(^{60}\) *Ibid.*, pp. 146 f. (Our notation.)
becomes scarcer with them, and $MU$, the marginal utility of money, rises.\textsuperscript{64}

This procedure is so similar to Marshall's that we may defer comment until we discuss the latter's more elaborate version.

Unlike Walras and Menger, Jevons considered the question of the interpersonal comparison of utilities. He expressly argued that this was impossible\textsuperscript{62} but made several such comparisons, as we shall notice later. Menger avoided the subject and did not engage in such comparisons; and Walras made only incidental interpersonal comparisons.\textsuperscript{63}

C. UTILITY MAXIMIZATION AND THE DEMAND CURVE

Menger simply ignored the relationship between utility and demand. He was content to set some demand prices (he worked always with discontinuous schedules) which somehow represented marginal utilities\textsuperscript{64} and proceeded to an elementary discussion of pricing under bilateral monopoly (the indeterminacy of which was recognized), duopoly (the complications of which were not recognized—a competitive solution was given), and competition (in which the absence of a theory of production had predictable effects).\textsuperscript{65}

Jevons' attempt to construct a bridge between utility and demand was seriously hampered, I suspect, by his inability to translate any but simple thoughts into mathematics. His fundamental equation for the maximization of utility in exchanges was presented as a fait accompli:

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}.$$  

This equation was satisfactory for an individual confronted by fixed prices, but how to apply it to competitive markets?

Jevons devised two concepts to reach the market analysis: the trading body and the law of indifference. A trading body was the large group of buyers or sellers of a commodity in a competitive market.\textsuperscript{66} The law of indifference was that there be only one price in a market.\textsuperscript{67}

He proceeded in the following peculiar manner. Let the equation of exchange be applied to each trading body; for each group of competitive individuals the equation will determine the relationship between the quantity offered and the quantity demanded.\textsuperscript{68}

Hence we have two equations to determine the two unknowns: the quantities

\textsuperscript{66}The requirement of competition was indirect: one characteristic of a perfect market was that "there must be no conspiracies for absorbing and holding supplies to produce unnatural ratios of exchange" (Theory [4th ed.], p. 86). It is evident that the trading body could not properly be used to explain prices, because its composition depended upon prices.

\textsuperscript{67}Jevons (ibid., p. 95) stated the law of indifference as

$$\frac{dx_1}{dx_2} = \frac{x_2}{x_1}.$$  

This notation is ambiguous (see Marshall, Memorials, p. 92; F. Y. Edgeworth, Mathematical Psychics [London: Paul, 1881], pp. 110 ff.).

\textsuperscript{68}Jevons seems to have introduced the trading bodies to get quickly to market prices, not because of an intuition that bilateral monopoly was indeterminate; at least he overlooked the difficulties in deeply (Theory [4th ed.], p. 117).
of \( x_1 \) and \( x_2 \) exchanged. Quite aside from the ambiguous concept of a trading body, this procedure was illicit on his own view that utilities of different individuals are not comparable.\(^{59}\)

Walras succeeded in establishing the correct relationship between utility and demand. He first derived the equations of maximum satisfaction for an individual: if there are \( m \) commodities, and a unit of commodity \( x_1 \) is the numéraire in terms of which the prices of other commodities are expressed (so \( p_1 = 1 \)), we have \((m - 1)\) equations:\(^{70}\)

\[
MU_1 = \frac{MU_2}{p_2} = \frac{MU_3}{p_3} = \ldots
\]

Finally, the budget equation states the equality of values of the initial stocks of commodities \((x_i^0)\) and the stocks held after exchange:

\[
x_1 + x_2 p_2 + x_3 p_3 + \ldots = x_1^0 + x_2^0 p_2 + x_3^0 p_3 + \ldots
\]

We thus have \( m \) equations to determine the \( m \) quantities of the commodities demanded or supplied by the individual. We may solve the equations for the quantities demanded or supplied as functions of the prices:

\[
x_2 = x_2(p_2, p_3, \ldots)
\]

\[
x_3 = x_3(p_2, p_3, \ldots)
\]

\[
\ldots \ldots \ldots \ldots
\]

\(^{59}\) "The reader will find, again, that there is never, in any single instance, an attempt made to compare the amount of feeling in one mind with that in another" (ibid., p. 14).

\(^{70}\) Éléments, Lesson 8. Let total utility = \( f(x_1) + g(x_2) + h(x_3) + \ldots \). In one of these utility functions, substitute the budget limitation,

\[
x_1 + x_2 p_2 + x_3 p_3 + \ldots = x_1^0 + x_2^0 p_2 + x_3^0 p_3 + \ldots,
\]

where \( x_1, x_2, x_3, \ldots \), are the initial stocks. Then maximize total utility to obtain the equations in the text.

\[
x_1 = (x_1^0 + x_2 p_2 + x_3^0 p_3 + \ldots)
\]

\[
- (x_2 p_2 + x_3 p_3 + \ldots).
\]

The \( x_1, x_2, x_3, \ldots \), are the quantities held (demanded), and \((x_2^0 - x_1), (x_3^0 - x_2), (x_3^0 - x_3), \ldots\) the quantities supplied.\(^{71}\)

To determine the market prices, we simply add the demands of all \( n \) individuals in the market for each commodity

\[
X_1 = \sum_1^n x_1 = \sum_2^n x_2(p_2, p_3, \ldots)
\]

\[
X_2 = \sum_3^n x_2 = \sum_{n} x_3(p_2, p_3, \ldots)
\]

\[
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\]

and equate the quantities demanded to the quantities available \((X_i^0)\):

\[
X_2^0 = X_2
\]

\[
X_3^0 = X_3
\]

\[
\ldots \ldots \ldots
\]

There are \((m - 1)\) such equations with which to determine the \((m - 1)\) prices of \( x_2, x_3, \ldots \), in terms of \( x_1 \). It may appear that we have forgotten the budget equation, but it is not an independent relationship because it can be deduced from the other equations. If we multiply the last set of equations by the respective prices of the commodities and add, we obtain

\[
p_2(X_2^0 - X_2) + p_3(X_3^0 - X_3) + \ldots = 0.
\]

But if we add the individual budget equations we obtain

\[
\sum x_i - x_i^0 = p_2(X_2^0 - X_2)
\]

\[
+ p_3(X_3^0 - X_3) + \ldots = 0.
\]

\(^{71}\) This summary differs in notation and detail, but not in substance, from Walras' exposition (ibid., pp. 113 ff.). The chief difference of detail is that Walras writes the utility as \( f(x_1 + x_2) \), where I write it as \( f(x_i) \), so his \( x_i \) can be negative.
Hence if the quantity demanded equals the quantity available in \((m - 1)\) markets, the equality must also hold in the \(m\)th market. This is equivalent to saying that if we know the amounts of \((m - 1)\) commodities that have been exchanged for each other and an \(m\)th commodity, and the rates of exchange, we necessarily know the amount of the \(m\)th commodity exchanged.

The (Walrasian) demand function is thus the relationship between the quantity of a commodity and all prices, when the individual’s (or individuals’) money income and tastes (utility functions) are held constant. We shall adhere to this meaning of the demand function or “curve” (the two-dimensional illustration of course requiring that all prices except that of the commodity are held constant), and the relationship between quantity and money income (all prices and tastes being held constant) will be designated as the income curve.

D. THE APPLICATIONS OF THE THEORY

Jevons gave only one application of his utility theory: a demonstration that both parties to an exchange gain satisfaction. The demonstration, as he gave it, was inconsistent with his denial of the possibility of comparing utilities of individuals, for it rested on the marginal utility curves of nations.\(^{72}\)

Menger was even less specific but surely vastly more persuasive in his applications of the theory: he made it the basis of economic theory. The theory was given many everyday illustrations (mostly hypothetical, to be sure): it explained exchange, the wages of textile workers during the Civil War cotton shortage, the shifts of goods between free and economic, etc. More important, the theory of production became simply an instance of the theory of marginal utility: productive services were distinguished from consumption services only in being goods of higher order. Menger’s version had no predictive value, nor did he conjecture any new economic relationships. Indeed at least two of the founders of marginal utility theory—Jevons was the exception—knew much less about economic life than a dozen predecessors such as Smith and Babbage. Yet the theory served to systematize a variety of known facts of everyday observation and seemed to confer an air of generality and structural elegance upon price theory.

Walras also did a good deal of this reorientation of economic theory in terms of utility, whereby the value of productive services was determined by the values of products. But he also attempted a specific and natural application of the theory to demand-curve analysis.

This application was the derivation of the law that price reductions will increase the quantity demanded; price increases will decrease the quantity demanded.\(^{73}\) Walras treated this as intuitively obvious, but it was a strict implication of his theory. Consider the equations of maximum satisfaction:

\[
\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \frac{MU_3}{p_3} = \ldots
\]

Assume \(p_2\) falls by \(\delta p_2\), and assume that the individual is deprived of his nominal increase in real income, \(2\delta p_3\). At the new price, \(p_2 - \delta p_2\), the individ-

\(^{72}\) *Theory* (4th ed.), pp. 144 ff. In the Preface to the second edition he proposed broader applications much closer to those of Menger and Walras but never worked out this position.

\(^{73}\) *Éléments*, pp. 131, 133.
ual obtains a larger marginal utility per dollar from $X_2$ than from other commodities, hence he will substitute $X_2$ for other commodities. Restore now the increment of income $x_2 \delta p_2$, and it will be used to purchase more of every commodity, including $x_2$. The individual necessarily buys more $X_2$ at a lower price, and therefore all individuals buy more of $X_2$ at a lower price: the demand curve for each product must have a negative slope.\footnote{The validity of this argument depends on the assumption that the marginal utility of a commodity is a (diminishing) function only of the quantity of that commodity (see Sec. IV).}

A second application of utility theory was made in the theorem on the distribution of stocks: a redistribution of initial stocks of goods among the individuals in a market, such that each individual's holdings have the same market value before and after the redistribution, will not affect prices.\footnote{Ibid., pp. 145–49.} It is the amount of income, not its composition in terms of goods, that influences consumer behavior. The most interesting point with respect to this obvious theorem is that Walras stopped here on the threshold of the analysis of the effects of income upon consumption. One may conjecture that his penchant for analyzing what are essentially barter problems in his theory of exchange played a large role in this failure to analyze income effects.\footnote{Perhaps mention should also be made of the applications of utility theory to labor. Jevons' theory of disutility was labored and at times confused (see my Production and Distribution Theories [New York: Macmillan, 1942], chap. ii). Walras' treatment was more elegant—he introduced the marginal utility of leisure in complete symmetry to the theory of consumption—but not much more instructive (Éléments, p. 202). Menger denied that labor was usually painful (op. cit., p. 149 n.).}

The theory of utility also led Walras\footnote{Marshall's theory of multiple equilibria is independent of utility analysis; it refers only to the long run, whereas Walras' theory is strictly short run. See Marshall, Pure Theory of Domestic Values ("London School Reprints" [London, 1930]).} to his theory of multiple equilibria.\footnote{Éléments, pp. 68–72; Wicksell restates the theory, Lectures on Political Economy (London: Macmillan, 1934), I, 55 ff.}

This theory deals with the exchange of one commodity for another in a competitive market, when both commodities have utility to the individual.\footnote{Ibid., pp. 145–49.} The possessors of $X_1$ have a fixed stock—how much will they offer at various prices of $X_1$ (in terms of $X_2$)? When $p_2$ is zero (no $X_2$ is given in exchange for a unit of $X_1$), they will naturally supply no $X_1$; the supply curve begins at (or above) the origin. At higher $p_2$, they will offer more $X_1$ to obtain more $X_2$, but beyond a certain price, $L$, further increases in the price of $X_1$ will lead them to reduce the quantity of $X_1$ offered because they become relatively sated with $X_2$. Walras illustrates this with Figure 2, where $D$ is the demand curve and $S$ the supply curve. $A'$ and $A''$ are points of stable equilibrium, be-
cause at higher prices the quantity supplied exceeds the quantity demanded and at lower prices the quantity demanded exceeds the quantity supplied. Point \( A \), however, is an unstable equilibrium because at higher prices the quantity demanded exceeds the quantity supplied so the price rises even more, and conversely at lower prices. We shall not follow the history of multiple equilibria, in which economists have usually taken an apprehensive pride.

In the area of welfare economics, Walras' most important application was the theorem on maximum satisfaction:

Production in a market governed by free competition is an operation by which the [productive] services may be combined in products of appropriate kind and quantity to give the greatest possible satisfaction of needs within the limits of the double condition that each service and each product have only one price in the market, at which supply and demand are equal, and that the prices of the products are equal to their costs of production.\(^{69}\)

This theorem, which is not true unless qualified in several respects, gave rise to an extensive literature which lies outside our scope.\(^{69}\)

\(^{69}\) Éléments, p. 231; Jevons also stated the theorem (Theory, 4th ed., p. 141).


IV. THE FORM OF THE UTILITY FUNCTION

The three founders of the utility theory treated the utility of a commodity as a function only of the quantity of that commodity. If \( x_1, x_2, x_3, \ldots \), are the commodities, the individual's total utility was written (explicitly by Jevons and Walras, implicitly by Menger), as

\[
J(x_1) + g(x_2) + h(x_3) + \ldots
\]

They further assumed that each commodity yielded diminishing marginal utility. This form of utility function has the implication that the demand curve for each commodity has a negative slope, as I have already remarked. It has also the implication that an increase in income will lead to increased purchases of every commodity. This is easily shown with the fundamental equations,

\[
MU_i = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \frac{MU_3}{P_3} = \ldots
\]

If income increases, the marginal utility of every commodity (and of income) must decrease, but the marginal utility of a commodity can be reduced only by increasing its quantity. This implication was not noticed.

Edgeworth destroyed this pleasant simplicity and specificity when he wrote the total utility function as \( \phi(x_1, x_2, x_3, \ldots) \). He appears to have made this change partly because it was mathematically more general, partly because it was congruent with introspection.\(^{61}\) The change had important implications for the measurability of utility that I shall discuss in Section V.

With the additive utility function, diminishing marginal utility was a sufficient condition for convexity of the in-
difference curves;\textsuperscript{83} with the generalized utility function, diminishing marginal utility was neither necessary nor sufficient for convex indifference curves.\textsuperscript{84} Nevertheless, Edgeworth unnecessarily continued to assume diminishing marginal utility, but he also postulated the convexity of the indifference curves.\textsuperscript{84}

Even with convexity, the generalized utility function no longer has the corollary that all income curves have positive slopes (or, therefore, that all demand curves have negative slopes).

\textsuperscript{85}Diminishing marginal utility for each commodity was not necessary; however, the indifference curves could be convex to the origin if every commodity except one yielded diminishing marginal utility, and the marginal utility of this exception commodity did not increase too rapidly. This exceptional case was first analyzed by Slutsky (see Sec. VII).

\textsuperscript{86}In the two-commodity case

\[
\frac{dx_2}{dx_1} = \frac{\varphi_1}{\varphi_2}
\]

is the slope of an indifference curve, and the condition for convexity is

\[
\frac{d^2x_2}{d^2x_1} = \frac{\varphi_1^2 - 2\varphi_{12}\varphi_{11} + \varphi_{22}}{\varphi_2^2} > 0
\]

where the subscripts to \(\varphi\) denote partial differentiation with respect to the indicated variables. It is clear that diminishing marginal utility (\(\varphi_{11}\) and \(\varphi_{22}\) negative) is not necessary for convexity, since \(\varphi_{11}\) can be positive and large, and it is not sufficient, since \(\varphi_{11}\) can be negative and large. In the additive case (\(\varphi_{12} = 0\)), at most one marginal utility can be increasing, as was pointed out in the previous footnote.

\textsuperscript{87}Mathematical Psychics, p. 36. He wrote the utility function as \(u(x_1, -x_2)\), in my notation, for reasons which will be pointed out below. He postulated that \(\varphi_{12} < 0\), where \(-x_2\) is work done by the person and \(X_1\) is remuneration received. This is equivalent to assuming that an increase in remuneration increases the marginal utility of leisure, and would be represented by \(\varphi_{12} > 0\) if we write the function as \(u(x_1, x_2)\), as is now customary. With diminishing marginal utility this condition leads to convexity (see previous note).

After a price reduction, \(\delta p_2\), we may again segregate the effect of a change in relative prices by temporarily reducing the individual's income by \(x_2\delta p_2\). When we restore this increment of real income, we cannot be sure that each commodity will be consumed in larger quantity. Suppose an increase in \(X_1\) reduces the marginal utility of \(X_2\). Then when a portion of the increment of real income \(x_2\delta p_2\) is spent on \(X_1\), \(MU_2\) may diminish so much that the amount of \(X_2\) must be reduced below its original quantity to fulfill the maximum satisfaction conditions.\textsuperscript{88}

The only further generalization of the utility function (aside from questions of measurability) was the inclusion of the quantities consumed by other people in the utility function of

\textsuperscript{89}The conditions for maximum satisfaction are

\[
\frac{\varphi_1}{\varphi_2} = \frac{p_1}{p_2},
\]

\(x_1p_1 + x_2p_2 = R\).

Differentiate these equations with respect to \(R\) (holding prices constant) and solve to obtain

\[
\frac{\partial x_2}{\partial R} = \frac{p_1\varphi_{11} - p_1\varphi_{12}}{p_2^2\varphi_{11} - 2p_1p_2\varphi_{12} + p_1^2\varphi_{22}}.
\]

The denominator of the right side is negative if the indifference curves are convex to the origin. The numerator, however, can be positive with \(\varphi_{12} < 0\), so the whole expression may be negative (\(X_1\) may be "inferior"). With the additive function, \(\varphi_{12} = 0\) (and of course they assumed \(\varphi_{11} < 0\)), so the expression must be positive (\(X_1\) and \(X_2\) must be "normal"). Similarly, differentiate the equations with respect to \(p_2\) holding \(p_1\) and \(R\) constant and solve to obtain

\[
\frac{\partial x_2}{\partial p_2} = \frac{p_1\varphi_{11} + x_1\varphi_{12} - x_2\varphi_{12}}{p_2^2\varphi_{11} - 2p_1p_2\varphi_{12} + p_1^2\varphi_{22}}.
\]

Again the denominator is negative, and the numerator may be negative if \(\varphi_{12}\) is negative, so the whole expression may be positive. With the additive utility function and diminishing marginal utility, the expression must be negative.
the individual. Thus one's pleasure from diamonds is reduced if many other people have them (or if none do), and one's pleasure from a given income is reduced if others' incomes rise. This line of thought is very old, but it was first introduced explicitly into utility analysis in 1892. Fisher casually suggested it:

Again we could treat utility as a function of the quantities of each commodity produced or consumed by all persons in the market. This becomes important when we consider a man in relation to the members of his family or consider articles of fashion as diamonds, also when we account for that (never thoroughly studied) interdependence, the division of labor.

Henry Cunynghame made the same suggestion more emphatically in the same year:

Almost the whole value of strawberries in March, to those who like this tasteless mode of ostentation, is the fact that others cannot get them. As my landlady once remarked, "Surely, sir, you would not like anything so common and cheap as a fresh herring?" The demand for diamonds, rubies, and sapphires is another example of this.

Pigou took up this argument, used it to show that consumer surpluses of various individuals cannot be added, but decided that these interrelationships of individuals' utilities were stable (and hence did not vitiate the consumer surplus apparatus) when the price changes were small. It was only proper that Marshall's leading pupil should postulate the constancy of the marginal utility of prestige.

Pigou's article elicited the first statistical investigation designed to test a utility theory (and apparently the only such investigation during the period). Edgeworth, a Fellow of All Souls, collected statistics from "a certain Oxford College" to determine "whether the size of the party has any influence upon the depth of the potations"—that is, upon the per capita consumption of wine. The data were presented in relative form lest they "should excite the envy of some and the contempt of others"; the conclusion was that the effect of the size of party was inappreciable.

A few subsequent attempts have been made to revive this extension of the utility function to include the effect on one person's utility of other people's consumption, but the main tradition has ignored the extension. This neglect seems to have stemmed partly from a belief in the unimportance of the effect and partly from the obstacles it would put in the way of drawing specific inferences from utility analysis.

There remain three subordinate topics that may conveniently be discussed here. They are (a) the graphical exposition of the theory of the generalized utility function; (b) the attitude of contemporary economists toward Edgeworth's generalization; and (c) the Bernoulli hypothesis on the shape of the utility function.

E.g.: A. Smith, Theory of Moral Sentiments (Boston: Wells & Lilly, 1817), Part III, chap. iii; Part IV, chap. i; N. F. Canard, Principes d'économie politique (Paris: Buisson, 1841), chap. vi; Senior, op. cit., p. 12.


"Some Remarks on Utility," Economic Journal, XIII (1903), 66 ff. He wrote the utility function of the individual as

\[ U = \phi (x, y, z, w, K (ab)) \]

where \( x, y, z, \) and \( w \) were quantities consumed by the individual, \( a \) was the quantity of \( x \) possessed by some other individual \( i \), whose social distance was \( b_i \), and \( K \) was a symbol "akin to, though not identical with, the ordinary \( \Sigma \)" (ibid., p. 61).

A. INDIFFERENCE CURVES

With the introduction of the interrelationship of utilities of commodities, it was no longer possible to portray total utility graphically in two dimensions. Edgeworth devised indifference curves, or contour lines, to permit of a graphical analysis of utility in this case. In itself this was merely an expositional advance, but it merits summarization because of its great popularity in modern times and because it later invited attention to questions relating to the measurability of utility.

We restrict ourselves to the case of two commodities, as Edgeworth and almost everyone since has done in graphical analysis. We define the indifference curve as the combinations of $X_1$ and $X_2$ yielding equal satisfaction, i.e., $\varphi(x_1, x_2) =$ constant. Edgeworth chose an asymmetrical graphical illustration of these curves that had a definite advantage for his purpose of analyzing bilateral monopoly. He let the abscissa represent the quantity of $X_1$ obtained by the individual, and the ordinate represent the quantity of $X_2$ given up.

It is evident that such indifference curves have a positive slope (if both commodities are desirable), for the individual will require more $X_1$ to offset (in utility) the loss of more $X_2$. In fact, the slope of the indifference curve with respect to the $X_1$ axis will be

$$\frac{dx_2}{dx_1} = \frac{MU_1}{MU_2}.$$  

In addition, Edgeworth postulated that

$\text{The three commodity indifference surfaces are of course the limit of literal graphical exposition, and even they have been deemed unappetizingly complex.}$

$\text{For } dx_1 MU_1 \text{ will be the gain of utility from an increment } dx_1, \text{ and } dx_2 MU_2 \text{ will be the loss of utility from a decrement } dx_2, \text{ and these must be equal if the movement is along an indifference curve.}$

The indifference curves are concave to the $X_1$ axis.

Edgeworth's pioneer demonstration of the indeterminacy of bilateral monopoly will illustrate the advantage of this formulation. A trader possessing $X_2$ but no $X_1$ would be at the origin; his indifference curves are those labeled $l$ in Figure 3. The second trader, who possesses $X_1$ but no $X_2$, will have the corresponding indifference curves (II), for he will be giving up $X_1$ and acquiring $X_2$ in exchange. The points where the two sets of indifference curves are tangent form a curve, CC, which Edgeworth christened the contract curve. The ends of the contract curve are determined by the condition that no trader be worse off after trading than before, i.e., by the indifference curves, $I_0$ and $II_0$. The final contract between the traders must take place on this contract curve, because if it occurred elsewhere, it would be to the gain of one party, and not to the loss of the other, to move to the curve. Thus point Q was not a tenable point of final contract because individual II can move from

$\text{Mathematical Psychics, pp. 20 ff.}$
II_1 to the higher indifference curve II_2, while I remains on the same indifference curve, I. Any point on the contract curve is a position of possible equilibrium, and the precise position reached will be governed by "higgling dodges and designing obstinacy, and other incalculable and often disreputable accidents."^94

Although this mode of exposition is convenient in the analysis of trade in two commodities between two individuals, it has no special advantage in the competitive case, and asymmetrical axes are awkward in algebraic analysis. Fisher introduced the now conventional graphical statement, in which the amounts held (or obtained) of the commodities appear on all axes.^95

**B. CONTEMPORARY PRACTICE**

Despite the intuitive appeal of Edgeworth's generalized utility function, economists adhered to the additive utility function with considerable tenacity. In the nonmathematical writings, such as those of Böhm-Bawerk, Wieser, and J. B. Clark, the additive function was used almost exclusively. Barone defended it as an approximation.^96 Wicksteed used it exclusively in his *Über Werte* (1894), although conceding the greater realism of the generalized function,^97 and found some place for it in his later *Lectures*.^98 Wicksteed used only the additive function in his *Alphabet* (1888)^99 and also in the elementary exposition of the theory in his *Common Sense* (1910) but not in the "advanced" statement.^100 Finally, Marshall and Pareto were so influential as to require more extended discussion.

Marshall also started with the Jevons-Walras assumption, to which he had probably arrived independently. This assumption was not explicit in the first edition of the *Principles* (1890), but one can cite evidence of its presence.

First, in his mathematical characterization of the utility function Marshall ignores any interdependence of utilities.^101 Second, he asserts the law of negatively sloping demand curves in all generality: "There is then one law and only one law which is common to all demand schedules, viz. that the greater the amount to be sold the smaller will be the price at which it will find purchasers."^102 This is a corollary of diminishing marginal utility only if the utility function is additive. Third, he was prepared to measure the utility of all commodities as the sum of the individual utilities: "We may regard the aggregate of the money measures of the total utility of wealth as a fair measure of that part of happiness which is dependent on wealth."^103

In the second edition (1891) the as-

^94 *Über Werte, Kapital und Renten* (Jena: Fischer, 1894), esp. p. 43.

^95 *Lectures on Political Economy*, I, 40–42, 55 ff.; however, the generalized function is preferred (*Ibid.* , pp. 41–42, 48–59, 79 ff.).


^98 *Principles of Economics* (London: Macmillan, 1890), Mathematical Notes II, III, VII [I, II, VI]. References in brackets will be used for corresponding passages in the eighth edition.


^100 *Ibid.*, pp. 179–80, also Mathematical Note VII. His Mathematical Note III [II] also implies an additive function if his p, "the price which [a person] is just willing to pay for an amount [x] of the commodity . . ." is interpreted as our *x*p, and the price to the person is treated as constant. See Sec. VII.
assumption became reasonably explicit:

Prof. Edgeworth's plan of representing $U$ and $V$ as general functions of $x$ and $y$ has great attractions to the mathematician; but it seems less adapted to express the everyday facts of economic life than that of regarding, as Jevons did, the marginal utilities of apples as functions of $x$ simply.

The facts both of everyday life and of contemporary theory soon led Marshall to make serious qualifications of his theory but never to qualify this statement.

Even in the first edition Marshall had inconsistently recognized the existence of "rival" products, which were defined as products able to satisfy the same desires. Fisher's discussion of competing and completing goods seems to have been the stimulus to Marshall to give more weight to interrelationships of utilities in the third edition of the *Principles* (1895). Once persuaded, Marshall modified his theory on two points. The first was that he slightly modified his assertion of the universality of negatively sloping demand curves and in fact introduced the Giffen paradox as an exception. The second alteration was in his treatment of consumers' surplus: "When the total utilities of two commodities which contribute to the same purpose are calculated on this plan, we cannot say that the total utility of the two together is equal to the sum of the total utilities of each separately." No important changes were made thereafter.

These alterations were only patchwork repairs; Marshall did not rework his theory of utility. He retained to the last a theory constructed on the assumption of an additive utility function.

Pareto also conceded the validity of the Edgeworth generalization but continued to use chiefly the additive function in his early work. Indeed, he offered the remarkable argument:

One sees now that instead of being able to use the indicated properties of the final degree of utility to demonstrate what laws demand and supply must obey, it is necessary to follow the opposite path, and use the knowledge of such laws one may obtain from experience to derive the properties of the final degree of utility. One cannot rigorously demonstrate the law of demand, but rather, from the directly observable fact that demand diminishes with the increase of price we deduce the consequence that the final degrees of utility may each be considered—as far as this phenomenon is concerned—as approximately dependent only on the quantity of the commodity to which it is related.

In the *Manuel*, however, he showed that the additive utility function leads to conclusions which are contradicted by experience, but defended it as an approximation which was permissible for large categories of expenditure and for small changes in the quantities of substitutes or complements. There is no reason to believe that this is true.

[to be concluded]
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