Analysis of An Eccentric Cam

FBD of follower

\[ \sum F^y = F_{23} - k(y + \delta) - m\ddot{y} = 0 \]

FBD of cam

\[ T = F_{23} e \sin \omega t = \left[ (ke + P) + (m\omega^2 - k)e \cos \omega t \right] e \sin \omega t \]

\[ = e(ke + P)e \sin \omega t + \frac{e^2}{2}(m\omega^2 - k)e \sin 2\omega t \]

\[ y = e - e\cos \omega t = e(1 - \cos \omega t) \]

\[ \dot{y} = e\omega \sin \omega t \]

\[ \ddot{y} = e\omega^2 \cos \omega t \]
a) Plot of Displacement, Velocity, Acceleration and Contact Force for an Eccentric Cam System

b) Graph of Torque Components and Total Cam-Shaft Torque

In the torque characteristic, the (+) area is equal to (-) area. i.e. Energy required to drive the follower in the forward direction is recovered when the follower returns. A flywheel may be used on the cam shaft to handle this fluctuating energy requirement.
About Jump

1. To prevent jump, increase the term $ke + P$, by increasing preload $P$, or spring constant $k$, or both.
2. Here, jump begins at $\omega t = 180^\circ$ with $\cos \omega t = -1$
3. Jump will not occur if, 

$$P > e(m\omega^2 - 2k)$$

Critical speed
Harmonic Motion

\[ y_{\text{rise}} = \frac{L}{2} \left(1 - \cos \frac{\pi \theta}{\beta} \right) \]
\[ \dot{y}_{\text{rise}} = \frac{\pi L}{2\beta} \left(\sin \frac{\pi \theta}{\beta} \right) \omega \]
\[ \ddot{y}_{\text{rise}} = \frac{\pi^2 L}{2\beta^2} \left(\cos \frac{\pi \theta}{\beta} \right) \omega^2 \]

\[ y_{\text{return}} = \frac{L}{2} \left(1 + \cos \frac{\pi \theta}{\beta} \right) \]
\[ \dot{y}_{\text{return}} = -\frac{\pi L}{2\beta} \left(\sin \frac{\pi \theta}{\beta} \right) \omega \]
\[ \ddot{y}_{\text{return}} = -\frac{\pi^2 L}{2\beta^2} \left(\cos \frac{\pi \theta}{\beta} \right) \omega^2 \]

\( \theta \) angle is measured from the beginning of rise, in rise formulations.

\( \theta \) angle is measured from the beginning of return, in return formulations.
For the first part of the parabolic rise ($\theta=0 \rightarrow \theta=\beta/2$):

$$y = A\theta^2 + B\theta + C$$
$$y' = \frac{dy}{d\theta} = 2A\theta + B$$
$$y'' = \frac{d^2y}{d\theta^2} = 2A$$

Boundary conditions: At $\theta=0$: $y=0$, $y'=0$ 
& at $\theta=\beta/2$: $y=L/2$ 
give,

$$A = \frac{2L}{\beta^2}$$

B=0, C=0. Therefore,

$$y = 2L\left(\frac{\theta}{\beta}\right)^2$$
$$y' = \frac{4L}{\beta^2}\theta$$
$$y'' = \frac{4L}{\beta^2} = \text{constant}$$
On the other hand, \[ \ddot{y} = \frac{dy}{dt} = \frac{dy}{d\theta} \frac{d\theta}{dt} = y' \omega \quad \text{and} \quad \ddot{y} = \frac{d^2y}{dt^2} = y'' \omega^2 + y' \alpha \]

For constant \( \omega \), \[ \ddot{y} = \frac{d^2y}{dt^2} = y'' \omega^2 \quad \ddot{y} = \frac{4L}{\beta^2} \omega^2 = \text{Constant} \]

For the second part of the parabolic rise (\( \theta = \beta/2 \longrightarrow \theta = \beta \) )

Boundary conditions: At \( \theta = \beta/2 \): \( y = L/2 \) & at \( \theta = \beta \): \( y = L \), \( \dot{y}' = 0 \)

\[ y = -2L \left( \frac{\theta}{\beta} \right)^2 + \frac{4L}{\beta} \theta - L \quad \dot{y}' = -\frac{4L}{\beta^2} \theta + \frac{4L}{\beta} \quad \dot{y}'' = -\frac{4L}{\beta^2} = \text{constant} \]

Therefore, \[ \ddot{y} = -\frac{4L}{\beta^2} \omega^2 = \text{Constant} \]
Problem: Parabolic Motion

Parabolic Rise 40 mm for $120^\circ$, Dwell for $30^\circ$, Parabolic Return for the remaining $210^\circ$

$k=5 \text{ kN/m}$, $P=35 \text{ N}$
$m=18 \text{ kg}$

a) Without computing numerical values, sketch approximate graphs of the displacement, acceleration and cam-contact force, versus cam angle. On this graph show where jump (lift-off) is most likely to begin.

b) At what cam speed would jump begin?