Structural Systems in Architecture
AR 361
Fall Semester

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Support Reactions, Determinacy, Stability Issues
Reactions

Statics of Structures - Reactions

Objectives

- Review of statics, identify the free body diagrams - use the equation of static equilibrium and equation of condition in the analysis of structures.
- Study support conditions and restraint.
- Calculate reactions for beams, frames and multi-story frames and trusses.
- Classify determinate and indeterminate structures.
- Determine if a structure is stable or unstable.
Introduction

A structure need to be able to support applied loads without changing shape, have large displacements or collapse. Even if in reality structures are not absolutely rigid, we will consider them as rigid bodies and we will base the analysis on the initial dimensions of the structure.
Deteminacy \[\rightarrow\] to complete the analysis of a structure only equation of equilibrium are enough. If the structure cannot be analyzed by the eqs. of statics it will be indeterminate.

Stability \[\rightarrow\] geometric arrangement of members and supports required to produce stable structures. If a structure is not able to support loads will have change in shape or large rigid-body displacements.
Supports

To ensure that a structure or a structural element remains in its required position, it is connected to the foundation or other structural member by supports. Support can be classified in function of restraints or reactions the support exert on the structure.
Supports (Continued)

- **Pin**
  - Prevent horizontal and vertical translation
  - Allow rotation

- **Hinge**
  - Prevent relative displacement
  - Allow rotation and displacements

- **Roller**
  - Prevent vertical translation
  - Allow horizontal translation and rotation

- **Rocker**

- **Elastomeric Pad**

**Supports**

- **R_{x}, R_{y}, e_{y}, R_{y}**
Supports (Continued)

**Fixed End**
- Prevent horizontal and vertical translation and rotation.

**Link**
- Prevent translation in the link direction.
- Allow translation parallel to the link and rotation.
- Prevent vertical translation and rotation.
- Allow horizontal translation.
Free Body Diagram

When we need to analyze a structure we need to draw the free body diagram of the structure with dimensions and all external and internal forces acting on the structure.
We can also cut a section and use the free body diagram to evaluate the internal forces.
Equations of Static Equilibrium

Once we define the free body diagram, we can use the equations of static equilibrium to evaluate the reactions.

\[
\begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma M_z &= 0
\end{align*}
\]

We can use the equations of equilibrium only for statically determinate structures.
Example 1

Compute the reaction of the beam.

The external force \( F = 40 \text{ kN} \) will have 2 components:

\[
F_x = 10 \times \frac{3}{5} = 6 \text{ kN}
\]

\[
F_y = 10 \times \frac{4}{5} = 8 \text{ kN}
\]
Example 1 (Continued)

EQUATION OF EQUILIBRIUM

\[ \sum F_x = 0 \]
\[ A_x + 6 = 0 \]
\[ A_x = -6 \text{ kN} \]

\[ \sum F_y = 0 \]
\[ A_y + B_y - 8 = 0 \]
\[ A_y = -4 \text{ kN} \]

\[ \sum M = 0 \]
\[ 10B_y - 8(15) = 0 \]
\[ B_y = 12 \text{ kN} \]
Example 2

Compute the reaction of the beam FAB as a link.

\[ F_{ABx} = 0.6 \times F_{AB} \]

\[ F_{ABy} = 0.8 \times F_{AB} \]
Example 2 - Continued

Equation of Equilibrium for the Link

\[ \sum F_x = 0 \quad F_A - F_B = 0 \quad F_A = F_B = F_{AB} \]

\[ \sum F_y = 0 \quad V_A - V_B = 0 \quad V_A = 0 \]

\[ \sum M_A = 0 \quad V_B \times s = 0 \quad V_B = 0 \]

Now we know which force is acting on BC \( \Rightarrow \) we can solve the beam.

\[ \sum F_x = 0 \quad 0.6F_{AB} - C_x = 0 \]

\[ C_x = 5.4 \text{ kN} \]

\[ \sum F_y = 0 \quad 0.8F_{AB} + C_y - 36 = 0 \]

\[ C_y = 28.8 \text{ kN} \]

\[ \sum M_C = 0 \quad 0.8F_{AB} (10) - 36 (2) = 0 \]

\[ F_{AB} = 9 \text{ kN} \]
Equations of Condition

If a structure is made of several rigid elements connected by a hinge or with other devices we need to divide the structure into several rigid bodies.

To solve this type of structures we need an extra equation called Equation of Condition or Equation of Construction.
Example 3

Compare the reaction for the beam. A load of 12 kN is directly applied to the hinge C.
Example 3 (Continued)

The supports provide 4 reactions. 3 equations of equilibrium are available and the hinge C provides one condition equation.

We can first solve the element C-E.

\[ 24 \times 5 = -E_y \times 10 \]

\[ E_y = \frac{24 \times 5}{10} = 12 \text{ kN} \]
Example 3 (Continued)

Considering \( E_y \) known, we can solve the all structure.

\[ \Sigma F_x = 0 \]

\[ E_x = 0 \]

\[ \Sigma F_y = 0 \]

\[ A_y + B_y + E_y - 12 - 24 = 0 \]

\[ \Sigma M_A = 0 \]

\[ -B_y \times 10 + 12 \times 15 + 24 \times 20 - E_y \times 25 = 0 \]

\[ B_y = \frac{1}{10} \left( 12 \times 15 + 24 \times 20 - 12 \times 25 \right) = 36 \text{ kN} \]

\[ A_y = 12 + 24 - 36 - 12 = -12 \text{ kN} \]
INFLUENCE OF REACTIONS ON STABILITY AND DETERMINACY OF STRUCTURES

To produce a stable structure, the designer must supply a set of supports that prevent the structure or any components from moving as a rigid body.

A structure needs to be stable and determinate.
Case 1:

For a rigid body to be in equilibrium, the 3 equations of equilibrium must be satisfied.

If we have less than 3 reactions the structure is UNSTABLE.

\[ ZF_y = 0 \quad R_1 + R_2 - P = 0 \]
\[ ZM_A = 0 \quad \frac{PL}{2} - R_2L = 0 \]
\[ ZF_x = 0 \quad \Theta = 0 \]

The structure is unstable because it will translate horizontally.
If we have a column that it is pinned support

\[ \sum F_x = 0 \quad R_x = 0 \]
\[ \sum F_y = 0 \quad R_y = P \]
\[ \sum M_A = 0 \quad 0 = 0 \]

The equations of equilibrium are satisfied and the structure is in equilibrium.
Case 1 (Continued)

The equation of equilibrium are satisfied and the structure is in equilibrium.

But the structure is unstable because the pin support does not prevent rotation.

\[ M = QL \]

\[ M = \Delta P \]

\[ R_x = Q \]

\[ R_y = P \]
A structure to be stable must have the capacity to resist load from any direction. To stabilize the column we can:

1. Replace the pin at A with a fixed support

2. We can add at point B a bracing system to avoid rotation, the member will not carry load but only align the column vertically.
Case 2: Supports supply 3 reactions $R_i = 3$

If the 3 equations of static equilibrium are satisfied the structure is in equilibrium.

If the equations are satisfied and the reactions are uniquely determinate the structure is externally determinate.

If a system of support supply 3 reaction but the equations of equilibrium cannot be satisfied the structure will be geometrically unstable.
Case 2 (Continued)

The member ABC carries a vertical load $P$ and a horizontal force $Q$.

Since all restraints are vertical, they offer no resistance to displacement in horizontal direction $\Rightarrow$ the reactions form a // force system.

From the equation of equilibrium, we have

$\sum F_x = 0 \quad Q = 0$ not consistent

Under the force $Q$, the structure will move until the link develops an horizontal component to equilibrate $Q$. 
The vertical and horizontal equilibrium is satisfied by the constraint but for the moment we have

$$\sum F_A = 0 \implies P_{ax} = 0$$

The lines of reaction are all passing through the pivot A and the reactions are equivalent to a concurrent force system.
Case 3: Restraint Greater Than 3 \( R > 3 \)

If the number of restraint is larger than 3 the structure will be indeterminate with degree of indeterminacy.

\[ DI = R - 3 \]

\( R \) = Number of Reactions

\[ DI = 8 - 3 = 5 \]
Case 3 (Continued)

Determinancy and stability of structures composed of several rigid bodies

If we indicate with \( R \) the internal restraint we will have:

1. If \( R < 3 + C \) STRUCTURE UNSTABLE

2. If \( R = 3 + C \) end reactions are not parallel or concurrent. The structure is stable. END DETERMINATE

3. If \( R > 3 + C \) end reactions are not parallel or concurrent. The structure is indeterminate

\[
DI = R - (3 + C)
\]
Example 4

Investigate the stability of the structure - Hinges at joints B and D.

As necessary condition for stability:

\[ R = 3 + c \]

\[ R = 5 \quad C = 2 \quad S = 3 + 2 \quad \text{OK} \]

The structure has many hinges and pins, so there is the possibility that the structure is geometrically unstable.
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