Investigation of critical phenomena and magnetism in amorphous Ising nanowire in the presence of transverse fields

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1. Introduction

The research on magnetic small particles is currently one of the most actively studied topics in statistical mechanics and condensed matter physics. The reason is due to the fact that these fine particles are considered as promising candidates in a wide variety of technological applications [1–7]. In particular, magnetic nanowires and nanotubes such as ZnO [8], FePt, and Fe3O4 [9] can be synthesized by various experimental techniques and they have many applications in nanotechnology [10,11], and they are also utilized as raw materials in fabrication of ultra-high density magnetic recording media [12–14]. Due to their reduced size, surface effects may become dominant, hence these nanoscaled small magnetic particles may exhibit several size dependent properties. For instance, it has been experimentally shown that the La0.67Ca0.33MnO3 (LCMN) nanoparticle exhibits a negative core–shell coupling, although the bulk LCMN is a ferromagnet [15,16].

From the theoretical point of view, these systems have been studied by a wide variety of techniques such as mean field theory (MFT) [17,18], effective field theory (EFT) [19–21], Green functions (GF) formalism [22], variational cumulant expansion (VCE) [23,24], and Monte Carlo (MC) simulations [25–31]. We learned from those studies that the core–shell concept can be successfully applied in nanomagnetism, since the aforementioned methods are capable of explaining various characteristic behaviors observed in nanoparticle magnetism. The MC simulation technique [32] is regarded as a powerful numerical approach for simulating the behavior of many complex systems, including magnetic nanoparticle systems. Nevertheless, the only disadvantage of this method is the need for a large amount of computer facilities due to
the long calculation times originating from the exhausting sampling averaging procedures. On the other hand, despite its mathematical simplicity, EFT is considered to be quite superior to conventional MFT since the former method exactly takes into account the single-site correlations and neglects the multi-site correlations whereas the latter technique ignores the whole correlations in the calculations. Therefore the results obtained by EFT are expected to be qualitatively more precise than those obtained by MFT. EFT formalism of ferroelectric nanoparticles has been introduced for the first time by Kaneyoshi [19]. In a series of consecutive studies, he extended the theory for the investigation of thermal and magnetic properties of the nanoscaled transverse Ising thin films [33], and also for cylindrical nanowire and nanotube systems [20,34].

Despite the growing technological advances, it is still hard to fabricate pure nanomaterials. The existence of disorder, such as site and bond disorder or the presence of random fields in magnetic nanoparticle systems constitutes an important role in material science, since it may induce some important macroscopic effects on the thermal and magnetic properties of real materials [35,36]. For instance, magnetic properties of CoFe$_2$O$_4$ particles with a magnetically disordered surface layer, and the surface spin disorder in NiFe$_2$O$_4$ nanoparticles have been investigated previously [37,38]. In addition, spin-glass surface disorder in antiferromagnetic small particles [17] has been examined. Based on EFT, the phase diagrams and thermal variation of the magnetization curves of a cylindrical nanowire with diluted surface sites have been investigated, and a number of characteristic phenomena have been found [39,40]. Besides, effects of the quenched disordered shell bonds, as well as interface bonds on the magnetic properties of the same model have been considered, and it has been found that a compensation point can be induced by a bond dilution process in the surface when the antiferromagnetic interface interactions are considered [41]. Very recently, the effect of the random magnetic field distribution on the phase diagrams and ground state magnetizations of the Ising nanowire has been investigated for discrete [42] and continuous random fields [43]. It has been found in these recent works that the system may exhibit reentrant behavior and first order phase transitions for discrete distribution of random fields which disappear for continuous random fields.

Apart from these, investigation of the effect of the structural disorder on the magnetism of nanoscaled particle systems with core–shell structure would be an interesting discussion, since the problem has not yet been examined in detail for nanoscaled magnetic particles, and the situation deserves particular attention. In discrete lattice models, the structural disorder is generally characterized by a random distribution of the nearest-neighbor interactions. In surface magnetism, in order to simulate an amorphous surface, the Handrich–Kaneyoshi model of amorphous ferromagnets [44] has been widely applied in the literature for thin films and semi-infinite ferromagnets within the framework of EFT. For amorphous Ising films [45–48] it has been found that the system may exhibit reentrant behavior, as well as first order transitions depending on the thickness of the film due to the presence of structural disorder and transverse fields whereas for semi-infinite ferromagnets [49,50], in addition to the observation of reentrant behavior on the surface originating from the existence of amorphization and transverse fields, it has also been observed that the reduced magnetization curve falls below those of the corresponding crystalline ferromagnets.

The purpose of the present work is to clarify within the framework of EFT, how the magnetism in a nanoparticle system is affected in the presence of an amorphous surface shell in the presence of transverse fields. In order to simulate a realistic system, we consider different transverse fields in the core and shell layers. The outline of the paper is organized as follows: in Section 2, we briefly present our model and the related formulation. The results and discussions are presented in Section 3, and finally Section 4 is devoted to our conclusions.

2. Formulation

We consider an infinitely long nanowire system composed of the usual Ising spins which are located on each lattice site (Fig. 1). The system is composed of a ferromagnetic core which is surrounded by a ferromagnetic shell layer. At the interface, we also define an exchange interaction between core and shell spins which can be of ferromagnetic type. The Hamiltonian

**Fig. 1.** Schematic representation of an infinitely long cylindrical Ising nanowire (top view). Sublattice magnetizations of the core layer are denoted by $m_1$ and $m_4$, whereas $m_2$ and $m_3$ define the sublattice magnetizations of the shell layer with the coordination numbers $z = 5$ and 6, respectively. Solid, dotted and dashed lines respectively represent the exchange interactions of the core, shell and interface.
equation describing our model can be written as

\[ \mathcal{H} = - \sum_{i\neq k} J_{\text{int}}^{(i,k)} \sigma_i^x \sigma_k^x - \sum_{i \neq j} J_{c}^{(i,j)} \sigma_i^x \sigma_j^x - \sum_{k \neq l} J_{\text{sh}}^{(k,l)} \sigma_k^x \sigma_l^x - \Omega_c \sum_i \sigma_i^z - \Omega_{\text{sh}} \sum_k \sigma_k^z, \]  

(1)

where \( \sigma = \pm 1 \). \( J_{\text{int}}, J_c, \) and \( J_{\text{sh}} \) define interface, core and shell coupling parameters, respectively. The terms \( \Omega_c \) and \( \Omega_{\text{sh}} \) represent the transverse fields acting on the core and shell sites, and the first three summations in Eq. (1) are carried out over the nearest-neighbor spins. For simplicity, we choose \( J_{\text{int}}^{(i,k)} = J_{\text{int}}, \) and \( J_{c}^{(i,j)} = J_c. \) However, in order to simulate an amorphous surface, we assume that the nearest-neighbor interactions in the shell layer are randomly distributed on the lattice according to the Handrich–Kaneyoshi probability distribution function [44]:

\[ P(j_{\text{sh}}^{(k,l)}) = \frac{1}{2} \delta(j_{\text{sh}}^{(k,l)} - J_{\text{sh}}(1 + \Delta)) + \frac{1}{2} \delta(j_{\text{sh}}^{(k,l)} - J_{\text{sh}}(1 - \Delta)), \]  

(2)

where \( \Delta \) is the structure factor. For \( \Delta = 0 \), the model reduces to pure system whereas for \( \Delta = 1 \) we obtain the bond dilution problem where half of the surface bonds are distributed as \( j_{\text{sh}}^{(k,l)} = 2J_{\text{sh}}, \) and for \( \Delta > 1 \) a surface frustration phenomenon takes place.

As shown in Fig. 1, we should divide both the ferromagnetic core and shell layers of the nanowire system into two sublattices. Namely, the lattice sites belonging to the outer boundary of the core layer are labeled by \( m_1 \) whereas the central site of the core is denoted by \( m_4. \) Similarly, for the shell layer of the system, we define \( m_2 \) and \( m_3 \) sublattices with the coordination numbers \( z = 5 \) and \( 6, \) respectively. Consequently, following the same methodology given in Ref. [39], we can get the usual coupled EFT equations for the magnetizations of core and shell layers of a nanowire system as follows

\[ m_1 = [A_1 + m_1 B_1]^4[A_3 + m_2 B_3][A_5 + m_3 B_5][A_7 + m_4 B_7][f_a(x)|_{x=0}, \]

\[ m_2 = [A_2 + m_2 B_2]^2[A_4 + m_3 B_4]^2[A_6 + m_4 B_6]^2[f_a(x)|_{x=0}, \]

\[ m_3 = [A_3 + m_3 B_3]^2[A_5 + m_4 B_5]^2[A_7 + m_4 B_7]^2[f_a(x)|_{x=0}, \]

\[ m_4 = [A_4 + m_4 B_4]^6[A_6 + m_4 B_6]^2[f_a(x)|_{x=0}, \]  

(3)

where \( m_1, m_2, m_3, m_4 \) terms denote the magnetizations of the core and shell sublattices, respectively. The coefficients in Eq. (3) are defined as follows

\[ A_1 = \cosh(J_c \nabla), \quad B_1 = \sinh(J_c \nabla), \]

\[ A_2 = \left( \cosh(j_{\text{sh}}^{(k,l)} \nabla) \right)_{i}, \quad B_2 = \left( \sinh(j_{\text{sh}}^{(k,l)} \nabla) \right)_{i}, \]

\[ A_3 = \cosh(J_{\text{int}} \nabla), \quad B_3 = \sinh(J_{\text{int}} \nabla), \]

(4)

with

\[ f_a(x) = \frac{x}{\sqrt{x^2 + \Omega_a^2}} \tanh \left[ \beta \sqrt{x^2 + \Omega_a^2} \right], \quad \alpha = c \ or \ s, \]

(5)

where \( \beta = 1/k_B T, k_B \) is the Boltzmann constant and \( T \) is the temperature. The random configurational averages \( \langle \ldots \rangle \) of surface exchange couplings in Eq. (4) can be obtained by using the probability distribution given in Eq. (2) and the relation \( \exp(\alpha \nabla) f_a(x + \alpha) = f_a(x + \alpha + \beta) \) [51,52].

With the help of a binomial expansion, Eq. (3) can be written in the form

\[ m_1 = \sum_{i=0}^{4} \sum_{j=0}^{1} \sum_{k=0}^{2} \sum_{l=0}^{2} K_1(i, j, k, l)m_1^i m_2^j m_3^k m_4^l, \]

\[ m_2 = \sum_{i=0}^{3} \sum_{j=0}^{1} \sum_{k=0}^{2} K_2(i, j, k)m_1^i m_2^j m_3^k, \]

\[ m_3 = \sum_{i=0}^{2} \sum_{j=0}^{1} \sum_{k=0}^{2} K_3(i, j, k)m_1^i m_2^j m_3^k, \]

\[ m_4 = \sum_{i=0}^{6} \sum_{j=0}^{3} K_4(i, j)m_1^i m_4^j, \]

(6)

where

\[ K_1(i, j, k, l) = \binom{4}{i} \binom{1}{j} \binom{2}{k} \binom{1}{l} A_1^{5-i-j-l} A_3^{4-j-k} A_5^{j-k} A_7^{l-k}, \]

\[ K_2(i, j, k) = \binom{1}{i} \binom{2}{j} \binom{2}{k} A_4^{4-j-k} A_3^{2} A_5^{j-k} A_7^{k}, \]
Since then a wire exhibits a strong ferromagnetic order in the core–shell interface, both core and shell layers contribute to parameter calculations, and all interactions and also the temperature of the system are normalized with ferromagnetic core coupling.

3. Results and discussion

$J$ means that the phasediagrams are independent of the sign of $\Omega$ effect of core transverse field on ferromagnetic systems. Interface interactions, and we have paid particular attention to the former case which simulates the behavior of semi-infinite ferromagnetic type. The magnetic properties of the system have been investigated in the presence of both weak and strong interface interactions, and we have paid particular attention to the former case which simulates the behavior of semi-infinite ferromagnetic systems. Finally, we should note that, as is discussed in Ref. [39], Eq. (10) is invariant under the transformation $J_{\text{int}} \rightarrow -J_{\text{int}}$ which means that the phase diagrams are independent of the sign of $J_{\text{int}}$.

In order to simulate a physically realistic system, different core and shell transverse fields are considered throughout the calculations, and all interactions and also the temperature of the system are normalized with ferromagnetic core coupling parameter $J_c$. We assume for the sake of simplicity that core, shell and core–shell interface exchange interactions are all ferromagnetic type. The magnetic properties of the system have been investigated in the presence of both weak and strong interface interactions, and we have paid particular attention to the former case which simulates the behavior of semi-infinite ferromagnetic systems.

In Fig. 2, we consider a cylindrical Ising nanowire with strong interface interactions in the absence of amorphization. The effect of core transverse field $\Omega_{c}/\Omega_{sh}$ on the phase diagrams in the $(k_B T_c/J_c - J_{sh}/J_c)$ plane when $\Omega_{sh}/J_c = 1.0$ is quite clear. Since the nanowire exhibits a strong ferromagnetic order in the core–shell interface, both core and shell layers contribute

$$K_3(i, j, k) = \left(\begin{array}{c} 2 \\ i \\ \end{array} \right) \left(\begin{array}{c} 2 \\ j \\ \end{array} \right) A_2^{i-j-k} A_2^{j-i} B_2^{i+k} B_2^j,$$

$$K_4(i, j) = \left(\begin{array}{c} 6 \\ i \\ \end{array} \right) A_4^{i-j} B_4^{i+j}.$$  

Once the coefficients given in Eq. (7) are calculated, we can construct a system of nonlinear coupled equations from Eq. (6) and obtain the magnetizations $m_i$, $i = 1, 2, 3, 4$. The longitudinal magnetizations of the core ($m_c$), shell ($m_{sh}$), and the total magnetization ($m_T$) of the system are defined as

$$m_c = \frac{1}{7} (6m_1 + m_4), \quad m_{sh} = \frac{1}{12} (6m_2 + 6m_3),$$

$$m_T = \frac{1}{19} (7m_c + 12m_{sh}).$$  

In the vicinity of the transition temperature, we have $m_i(T \to T_c) \simeq 0$. Hence, in order to obtain the critical temperature we may linearize Eq. (6), i.e.,

$$Am = 0,$$

where

$$A = \begin{pmatrix} K_1(1, 0, 0, 0) - 1 & K_1(0, 1, 0, 0) & K_1(0, 0, 1, 0) & K_1(0, 0, 0, 1) \\ K_2(1, 0, 0) & K_2(0, 1, 0) - 1 & K_2(0, 0, 1) & 0 \\ K_3(1, 0, 0) & K_3(0, 1, 0) & K_3(0, 0, 1) - 1 & 0 \\ K_4(1, 0, 0) & 0 & 0 & K_4(0, 1, 0) - 1 \end{pmatrix},$$

and

$$m = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}.$$  

For the selected values of the system parameters, the transition temperature of the system can be obtained from $\det(A) = 0$. Finally, we should note that, as is discussed in Ref. [39], Eq. (10) is invariant under the transformation $J_{\text{int}} \rightarrow -J_{\text{int}}$ which means that the phase diagrams are independent of the sign of $J_{\text{int}}$. 

![Fig. 2. Phase diagram of the system in a $(k_B T_c/J_c - J_{sh}/J_c)$ plane for the system parameters $J_{\text{int}}/J_c = 1.0$, $\Omega_{sh}/J_c = 1.0$, $\Delta = 0.0$, and for various selected $\Omega_{c}/\Omega_{sh}$ values.](image-url)
to the transition temperature. Namely, as the temperature increases starting from zero, the shell layer undergoes a second order phase transition when the core does. As the core transverse field increases, the transition temperature decreases, and the variation of core transverse fields does not affect the shell magnetization, even if the transition temperature does not change. This situation is also depicted for weak core transverse fields in Fig. 4.

The critical temperature of the system is determined by the shell layer. Hence, the variation of core transverse fields does not affect the shell magnetization. Therefore, the variation of the shell magnetization, even if the transition temperature does not change, is shown in Fig. 5(a) for the curves with $\Omega_{m}/\Omega_{sh}$ = 0.0 in Fig. 4. As another example, let us discuss the magnetization behavior corresponding to the shell layer for weak core transverse fields, as shown in Fig. 4. In this case, the critical temperature of the system is determined by the shell magnetization. Hence, the variation of core transverse fields does not affect the shell magnetization.
Fig. 4. Phase diagrams of the system in a \((k_B T / J_c - J_{sh}/J_c)\) plane with \(J_{int}/J_c = 0.01, \Omega_{sh}/J_c = 1.0\) and for various \(\Omega_c/\Omega_{sh}\) values. The value of amorphization parameter in the panels is selected: (a) \(\Delta = 0.0\), (b) \(\Delta = 0.5\), (c) \(\Delta = 1.0\), and (d) \(\Delta = 1.1\).

Fig. 5. (a) Temperature dependence of total magnetization \(m_T\) as a function of \(J_{sh}/J_c\) corresponding to Fig. 4(b) for some selected system parameters. (b) Ground state magnetization versus \(J_{sh}/J_c\) curves for given system parameters with \(\Delta = 0.0, 0.5, 1.0,\) and 1.1. (c) Variation of \(m_T\) with temperature as a function \(\Omega_c/\Omega_{sh}\) corresponding to Fig. 4 for a given set of parameters. (d) Ground state magnetization versus \(\Omega_c/\Omega_{sh}\) curves for \(\Delta = 0.0, 0.5, 1.0,\) and 1.1 with some selected system parameters.

the behavior of shell magnetization due to the existence of weak ferromagnetic interface interactions \((J_{int}/J_c = 0.01)\). Consequently, the transition temperature of the overall system is independent of \(\Omega_c/\Omega_{sh}\) values. A typical example of thermal variation of \(m_T\) curves as a function of core transverse field \(\Omega_c/\Omega_{sh}\) with the system parameters \(J_{int}/J_c = 0.01, J_{sh}/J_c = 5.0, \Omega_{sh}/J_c = 1.0,\) and \(\Delta = 0.5\) corresponding to Fig. 4(b) is represented in Fig. 5(c). As shown in this figure, the ground state value of total magnetization decreases with increasing \(\Omega_c/\Omega_{sh}\) values. However, for sufficiently large values of \(\Omega_c/\Omega_{sh}\), we expect that spins of the core layer are always directed perpendicular to the \(z\)-direction. Therefore, after this critical value of \(\Omega_c/\Omega_{sh}\), the core layer cannot exhibit a ferromagnetic order, and according to Eq. (8), the ground state value of \(m_T\) is determined by \(m_T = 12\Omega_{sh}/19\) which depends on the values of shell transverse field and amorphization parameter. Namely, in the case of \(\Omega_{sh} = \Delta = 0.0,\) we get \(m_T = 12/19\). In Fig. 5(d), we depict the variation of the ground state value of
\[ \Delta = \Omega \]

\[ \text{affect the core magnetization, since there exists a weak ferromagnetic coupling} \]

\[ \text{weakly amorphous surfaces} \]

\[ \text{the common features of the system for pure case} \]

\[ \text{decreases continuously, and it reduces to zero at a certain} \]

\[ \text{effect of disorder on the shell layer (i.e., the} \]

\[ \text{We have also examined the temperature dependence of total magnetization} \]

\[ m_T \text{ curves as a function of core transverse field} \]

\[ \Omega_c / \Omega_{sh} \]

\[ \text{magnetization, weakly amorphous surfaces} \text{ are inherently slided to highervalues. These are the common features of the system for pure case} (\Delta = 0.0), \text{as well as for weakly amorphous surfaces} (\Delta = 0.5), \text{surface bond dilution} (\Delta = 1.0), \text{and surface frustration} (\Delta = 1.1) \text{ phenomena. By comparing Fig. 6(a)−(d), it can be clearly observed that as} \Delta \text{ increases, shell magnetization weakens and the ferromagnetic region in the phase diagrams gets narrower. However, varying values of} \Delta \text{ does not affect the transition temperature unless it is determined by shell magnetization.} \]

\[ \text{We have also examined the temperature dependence of total magnetization} m_T \text{ as a function of shell transverse field} \]

\[ \Omega_{sh} / \Omega_c \]

\[ \text{when the transition temperature is determined by core magnetization. As it has been mentioned above, in this case this transition temperature of the system is independent of} \]

\[ \Omega_{sh} / \Omega_c, \text{however ground state behavior of} m_T \text{ curves explicitly depends on the value of} \Omega_{sh} / \Omega_c \text{. A typical example is depicted in Fig. 7(a) corresponding to a horizontal line in Fig. 6 with} \]

\[ \Delta = 0.5 \text{ and} \]
Fig. 7. (a) Temperature dependence of total magnetization $m_T$ as a function of surface transverse field $\Omega_{sh}/\Omega_c$ corresponding to Fig. 6(b) with $\Delta = 0.5$. (b) Ground state magnetization versus $\Omega_{sh}/\Omega_c$ plots for given system parameters with $\Delta = 0.0, 0.5, 1.0$, and 1.1.

When the value of $\Omega_{sh}/\Omega_c$ becomes sufficiently large, the shell layer cannot exhibit a ferromagnetic order within the whole temperature range. Therefore, ground state magnetization is determined from Eq. (8) as $m_T = 7m_c/19$. The situation is also depicted in Fig. 7(b) for some selected values of $\Delta$. Based on the results presented in Fig. 7, ground state values of $m_T$ curves exhibit a monotonic decreasing behavior with increasing $\Omega_{sh}/\Omega_c$ values for $0.0 \leq \Delta \leq 1.0$. In the presence of surface frustration (see the $\Delta = 1.1$ curve in Fig. 7(b)), $m_T$ curves exhibit highly non-monotonous and exotic ground states due to the thermal behavior of shell magnetization. This interesting behavior signals the occurrence of reentrant phenomena on the shell layer which can be regarded as a characteristic feature of the system due to the frustration.

In order to discuss in detail how the magnetic properties of the nanowire system are affected in the presence of surface amorphization, the phase diagrams in the $(k_B T_c / J_c - \Delta)$ plane are depicted in Fig. 8 with a variety of shell transverse field values, and for four typical values of ferromagnetic shell coupling $J_{sh}/J_c$ by considering a weak ferromagnetic coupling on the core–shell interface when $\Omega_{sh}/\Omega_c = 1.0$. Based on this figure, it is clear that the transition temperature of the system is determined by the shell magnetization for weak $\Delta$ values. However, as the value of structural disorder increases...
then the shell magnetization weakens and reduces to zero at a certain $\Delta$ value. Hereafter, contribution to the transition temperature completely comes from the core magnetization. By comparing Fig. 8(a)–(d) with each other, we can also claim that the ferromagnetic region in the phase diagrams in the $(k_BT/E_c - \Delta)$ plane gets expanded with increasing $J_{sh}/J_c$, whereas it becomes narrower with increasing $\Omega_{sh}/\Omega_c$, but does not vanish, since the variation of $\Omega_{sh}/\Omega_c$ does not affect the magnetization of the core layer.

According to Fig. 8, for weak $J_{sh}/J_c$ values, the transition temperature is not affected by $\Omega_{sh}/\Omega_c$ and $\Delta$ variations. This situation is illustrated in Fig. 9. In Fig. 9(a), we show the variation of total magnetization $m_T$ with temperature as a function of $\Omega_{sh}/\Omega_c$ for a weak interface $(J_{int}/J_c = 0.01)$, and relatively moderate shell $(J_{sh}/J_c = 1.0)$ couplings. As shown in this figure, only the ground state magnetization values are affected from varying $\Omega_{sh}/\Omega_c$ values. The mechanism underlying this fact has just been explained above (cf. explanations of Fig. 7), hence we will not discuss them again. On the other hand, variation of ground state values of $m_T$ curves with structural surface disorder $\Delta$ is depicted in Fig. 9(b) for some selected $\Omega_{sh}/\Omega_c$ values. For $\Omega_{sh}/\Omega_c = 0.0$, the $m_T$ – $\Delta$ curve exhibits three plateaus. Namely, according to our calculations, the first plateau appears at $m_T = 0.993$ for $0 \leq \Delta < 1.0$, the middle one is observed at $m_T = 0.662$ within the interval $1.0 \leq \Delta < 2.0$, and the last plateau is located at $m_T = 0.528$ for $\Delta \geq 3.0$. This plateau mechanism originates from the shell magnetization of the nanowire system. These three distinct ferromagnetic plateaus tend to disappear with increasing $\Omega_{sh}/\Omega_c$ values.

As a final investigation, let us discuss the conditions for the occurrence of reentrant phenomena in the system by investigating the thermal behavior of magnetization curves of the nanowire system. In Fig. 10, we represent projections of core, shell and total magnetization curves on a $(\Delta - k_BT/E_c)$ plane corresponding to Fig. 8(b) with $\Omega_{sh}/\Omega_c = 0.0$. At first glance, we can claim according to Fig. 10(a) that amorphization of the shell layer does not have a prominent effect on the thermal behavior of core magnetization whereas according to Fig. 10(b), shell magnetization does not exhibit reentrant behavior even in the presence of frustration ($\Delta > 1.0$). Moreover, as shown in the inset in Fig. 10(b), when the frustration exists in the shell layer, shell magnetization exhibits a partially ordered phase at low temperatures. Furthermore, variation of total magnetization is plotted in Fig. 10(c). We can clearly see that total magnetization always exhibit a second order phase transition between ferromagnetic and paramagnetic phases for the whole range of $\Delta$ values. As shown in Fig. 11, when the interface coupling between the core–shell frontier is zero, then the core and shell layers become completely independent of each other and accordingly, the partially ordered phase observed in the shell magnetization for weak interface coupling $J_{int}/J_c = 0.01$ (cf. Figs. 10(c) and 11(b)) disappears and the shell layer exhibits a reentrant behavior of second order which tends to disappear with increasing $\Delta$ values. On the other hand, in order to see how the ferromagnetic exchange coupling $J_{sh}/J_c$ of the shell layer affects the reentrant phenomena, we plot the temperature dependence of core, shell and total magnetization curves for relatively moderate $(J_{sh}/J_c = 3.0)$, and quite large $(J_{sh}/J_c = 10.0)$ values of shell coupling parameter for $J_{int}/J_c = 0.01$ in Fig. 12(a) and (b), respectively. By comparing Fig. 12(a) and (b) with each other we see that variation of $J_{sh}$ has no effect on the thermal variation of the core layer. However, magnetization of the shell layer, as well as total magnetization of the nanowire system exhibit a double reentrant behavior for large values of $J_{sh}/J_c$ where we observe three successive second order phase transitions. Similar observations have also been reported for Ising thin film [45] and also for semi–infinite ferromagnetic systems [50]. Finally, for the completeness of the work it would be interesting to investigate the effect of both core and shell transverse fields on the reentrant phenomena. For this purpose we take the results presented in Fig. 12(a) as a basis, and vary the transverse field values. In this context, Fig. 13(a) shows that if the core transverse field $\Omega_c/J_c$ is sufficiently large, such as $\Omega_c/J_c = 5.0$ then it becomes dominant against the exchange interactions, and a competition takes place between transverse field $\Omega_c/J_c$ of the core and amorphization parameter $\Delta$, hence both core and shell magnetizations exhibit reentrant behavior. On the other hand, in order to see whether the system exhibits a reentrant behavior due to the existence of the shell transverse field $\Omega_{sh}/\Omega_c$ or not, we present related results in Fig. 13(b). Namely, if one compares Figs. 12(a) and 13(b) with each other it can be seen that increasing $\Omega_{sh}/\Omega_c$ values causes a decline in the shell magnetization, however it is not sufficient to observe reentrence phenomena.
Fig. 10. Projection of (a) core magnetization $m_c$, (b) shell magnetization $m_{sh}$, and (c) total magnetization $m_T$ of the nanowire on a $(\Delta - k_B T/J_c)$ plane corresponding to Fig. 8(b) with $\Omega_{sh}/\Omega_c = 0.0$.

Fig. 11. Temperature dependence of shell magnetization ($m_{sh}$) curves as a function of amorphization parameter $\Delta$ with some selected system parameters for (a) $J_{int}/J_c = 0.01$, and (b) $J_{int}/J_c = 0.0$. 
4. Conclusions

In conclusion, we have discussed the magnetic properties of a cylindrical Ising nanowire in the presence of surface shell amorphization by considering ferromagnetic exchange couplings. In order to simulate a physically realistic model, we have presented the results for a system where the core and shell transverse fields are different from each other. In particular, we have focused our attention on the system where core and shell spins interact through a weak ferromagnetic interface coupling such as $J_{int}/J_c = 0.01$, and under these circumstances, we have found that the system exhibits similar characteristic features with that observed for Ising thin films, and semi-infinite ferromagnets in the presence of surface amorphization. Namely, according to the phase diagrams plotted in $(k_B T_c/J_c - J_{sh}/J_c)$ and $(k_B T_c/J_c - \Delta)$ planes, it has been found that depending on the values of system parameters, a shell layer may order ferromagnetically even if the core layer is paramagnetic (and vice versa) which means that the transition temperature of shell magnetization is greater than that of the core. This phenomenon is known as an extraordinary phase transition [49,50] and it is a general feature of finite magnetic systems where the surface effects are prominent. In the subsequent discussions, we have clarified the effect of the surface shell amorphization on the aforementioned features of the system, and we have observed that an Ising nanowire with an amorphous surface shell may exhibit magnetization curves which fall below those of the corresponding crystalline counterparts with increasing temperature which is generally observed in other types of amorphous ferromagnets [44].

Furthermore, we have investigated the ground state behavior of total magnetization $m_T$ curves and we have observed that $m_T$ curves exhibit highly non-monotonous and exotic ground states in the presence of frustration in the surface shell. We have also analyzed the necessary conditions for the occurrence of reentrant behavior in the system and we have obtained the following results:

- In the presence of frustrated surface shell ($\Delta > 1.0$) and weak core--shell interface coupling ($J_{int}/J_c = 0.01$), the shell magnetization exhibits a partially ordered phase in the $k_B T_c/J_c - \Delta$ plane (cf. see Fig. 10).
- If the ferromagnetic interaction in the core--shell frontier is zero (e.g. $J_{int}/J_c = 0.0$), this partially ordered phase disappears and we can observe a reentrance in shell magnetization.
- As previously mentioned in Ref. [50] for semi-infinite ferromagnets, it is very difficult to observe the surface reentrant phenomenon in these types of finite systems, since the $J_{sh}$ must be taken as a rather large value. This statement fits well...
for the observations presented in Fig. 12 of this work where we observe double reentrance for values of the shell coupling $J_{sh}$ that are ten times greater than the core coupling $J_c$.

- If the core transverse field $\Omega_c / J_c$ is sufficiently large, both core and shell magnetizations exhibit reentrant behavior, due to a competition between core transverse field $\Omega_c / J_c$ and structural disorder parameter $\Delta$.

- Finally, our results indicate that the existence of reentrant phenomena cannot be directly related to the variation of shell transverse field.

Based on the physical observations mentioned above, it would be interesting to perform calculations to analyze compensation phenomena in the amorphous Ising nanowire by considering antiferromagnetic core–shell interface ($J_{int} / J_c < 0$). However, this may be the subject of a future work.

Acknowledgments

One of the authors (YY) would like to thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for partial financial support. The numerical calculations reported in this paper were performed at TÜBİTAK ULAKBİM, High Performance and Grid Computing Center (TR-Grid e-Infrastructure).

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