KINEMATICS OF RIGID BODIES
Introduction

In rigid body kinematics, we use the relationships governing the displacement, velocity and acceleration, but must also account for the rotational motion of the body. Description of the motion of rigid bodies is important for two reasons:

1) To generate, transmit or control motions by using cams, gears and linkages of various types and analyze the displacement, velocity and acceleration of the motion to determine the design geometry of the mechanical parts. Furthermore, as a result of the motion generated, forces may be developed which must be accounted for in the design of the parts.

2) To determine the motion of a rigid body caused by the forces applied to it. Calculation of the motion of a rocket under the influence of its thrust and gravitational attraction is an example of such a problem.
A rigid body is a system of particles for which the distances between the particles and the angle between the lines remain unchanged. Thus, if each particle of such a body is located by a position vector from reference axes attached to and rotating with the body, there will be no change in any position vector as measured from these axes. Of course this is an idealization since all solid materials change shape to some extent when forces are applied to them.
Nevertheless, if the movements associated with the changes in shape are very small compared with the movements of the body as a whole, then the assumption of rigidity is usually acceptable.

For example, the displacements due to the flutter of an aircraft wing do not affect the description of the aircraft as a whole and thus the rigid body assumption is acceptable.

On the other hand, if the problem is one of describing, as a function of time, the internal wing stress due to wing flutter, then the relative motions of portions of the wing cannot be neglected, and the wing may not be considered as a rigid body.
Plane Motion

A rigid body executes plane motion when all parts of the body move in parallel planes. The plane of motion is considered, for convenience, to be the plane which contains the mass center, and we treat the body as a thin slab whose motion is confined to the plane of the slab. This idealization adequately describes a very large category of rigid body motions encountered in engineering.

The plane motion of a rigid body is divided into several categories:
Translation

It is any motion in which every line in the body remains parallel to its original position at all times. In translation, there is no rotation of any line in the body.

In **rectilinear translation**, all points in the body move in parallel straight lines.

Rocket test sled
In **curvilinear translation**, all points move on congruent curves.

In each of the two cases of translation, the motion of the body is completely specified by the motion of any point in the body, since all the points have the same motion.
Rotation about a fixed axis is the angular motion about the axis. All particles in a rigid body move in circular paths about the axis of rotation and all lines in the body which are perpendicular to the axis of rotation rotate through the same angle at the same time.
General Plane Motion

It is the combination of translation and rotation.
Plane motion = Translation with \( A \) + Rotation about \( A \)

\[ (a) \]

Plane motion = Translation with \( B \) + Rotation about \( B \)
We should note that in each of the examples cited, the actual paths of all particles in the body are projected onto the single plane of motion.

Analysis of the plane motion of rigid bodies is accomplished either by directly calculating the absolute displacements and their time derivatives from the geometry involved or by utilizing the principles of relative motion.
The rotation of a rigid body is described by its angular motion. The figure shows a rigid body which is rotating as it undergoes plane motion in the plane of the figure. The angular positions of any two lines 1 and 2 attached to the body are specified by $\theta_1$ and $\theta_2$ measured from any convenient fixed reference direction.

Because the angle $\beta$ is invariant, the relation $\theta_2 = \theta_1 + \beta$ upon differentiation with respect to time gives $\dot{\theta}_2 = \dot{\theta}_1$ and $\ddot{\theta}_2 = \ddot{\theta}_1$. Or, during a finite interval, $\Delta \dot{\theta}_2 = \Delta \dot{\theta}_1$. Thus, all lines on a rigid body in its plane of motion have the same angular displacement, the same angular velocity and the same angular acceleration.
The angular motion of a line depends only on its angular position with respect to any arbitrary fixed reference and on the time derivatives of the displacement. Angular motion does not require the presence of a fixed axis, normal to the plane of motion, about which the line and the body rotate.
Angular Motion Relations

The angular velocity $\omega$ and angular acceleration $\alpha$ of a rigid body in plane rotation are, respectively, the first and second time derivatives of the angular position coordinate $\theta$ of any line in the plane of motion of the body. These definitions give

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega d\omega = \alpha d\theta \quad \text{or} \quad \dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta$$

In each of these relations, the positive direction for $\omega$ and $\alpha$, clockwise or counterclockwise, is the same as that chosen for $\theta$. 
For rotation with **constant angular acceleration**, the relationships become

\[
\begin{align*}
\omega &= \omega_0 + \alpha t \\
\omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \\
\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2
\end{align*}
\]

Here \( \theta_0 \) and \( \omega_0 \) are the values of the angular position coordinate and angular velocity at time \( t = 0 \) and \( t \) is the duration of the motion considered.

As seen, the relationships given for the rotary motion of rigid bodies are analogous to those derived for the particle.
Rotation About a Fixed Axis

When a rigid body rotates about a fixed axis, all points other than those on the axis move in concentric circles about the fixed axis. Thus, for the rigid body in the figure rotating about a fixed axis normal to the plane of the figure through $O$, any point such as $A$ moves in a circle of radius $r$. So the velocity and the acceleration of point $A$ can be written as

\[v = r \omega\]

\[a_n = r \omega^2 = \frac{v^2}{r} = v \omega\]

\[a_t = r \alpha\]
These quantities may be expressed alternatively using the cross product relationship of vector notation. The vector formulation is especially important in the analysis of three dimensional motion. The angular velocity of the rotating body may be expressed by the vector $\vec{\omega}$ normal to the plane of rotation and having a sense governed by the right hand rule. From the definition of the vector cross product, the vector $\vec{V}$ is obtained by crossing $\vec{\omega}$ into $\vec{r}$. This cross product gives the correct magnitude and direction for $\vec{V}$.

$$\vec{V} = \dot{\vec{r}} = \vec{\omega} \times \vec{r}$$

The order of the vectors to be crossed must be retained. The reverse order gives

$$\vec{r} \times \vec{\omega} = -\vec{V}$$
The acceleration of point \( A \) is obtained by differentiating the cross product expression for \( \vec{v} \), which gives

\[
\vec{a} = \dot{\vec{v}} = \vec{\omega} \times \dot{\vec{r}} + \dot{\vec{\omega}} \times \vec{r}
\]

\[
= \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r}
\]

\[
= \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}
\]

Here \( \vec{\alpha} = \dot{\vec{\omega}} \) stands for the angular acceleration of the body. Thus, we can write

\[
\vec{v} = \vec{\omega} \times \vec{r}
\]

\[
\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})
\]

\[
\vec{a}_t = \vec{\alpha} \times \vec{r}
\]
For three dimensional motion of a rigid body, the angular velocity vector $\vec{\omega}$ may change direction as well as magnitude, and in this case, the angular acceleration, which is the time derivative of angular velocity, $\vec{\alpha} = \dot{\vec{\omega}}$, will no longer be in the same direction as $\vec{\omega}$. 
1. The angular velocity of a gear is controlled according to \( \omega = 12 - 3t^2 \), where \( \omega \) in rad/s is positive in the clockwise sense and where \( t \) is the time in seconds. Find the net angular displacement \( \Delta \theta \) from the time \( t = 0 \) to \( t = 3 \) s. Also find the total number of revolutions \( N \) through which the gear turns during the three seconds.

\[
\frac{d\omega}{dt} = \alpha = -6t \quad \quad \frac{d\theta}{dt} = \omega \quad \quad d\theta = \omega \, dt
\]

\[
\int_0^\theta d\theta = \int_0^3 (12 - 3t^2) \, dt
\]

\[
\theta = 12t - \frac{3}{3} t^3 \bigg|_0^3 = 12(3) - 3^3 = 9 \text{ rad}
\]

\( \Delta \theta = 9 \text{ rad} \)
Does the gear stop between \( t = 0 \) and \( t = 3 \) seconds?

\[
\omega = 12 - 3t^2 = 0 \quad \text{and} \quad 12 = 3t^2 \quad t = 2 \text{ s} \quad \text{(it stops at } t = 2 \text{ s)}
\]

\[
\int_0^{\theta_1} d\theta = \int_0^2 (12 - 3t^2) \, dt \quad , \quad \theta_1 = 12t - \frac{3}{3}t^3 \bigg|_0^2 = 12(2) - 2^3 = 16 \text{ rad}
\]

\[
\int_0^{\theta_2} d\theta = \int_2^3 (12 - 3t^2) \, dt \quad , \quad \theta_2 = 12t - \frac{3}{3}t^3 \bigg|_2^3 = -7 \text{ rad}
\]

\[16 + |-7| = 23 \text{ rad}\]

\[1 \text{ revolution} = 2\pi \text{ rad}\]

\[N \text{ revolutions} = 23 \text{ rad} , \quad N = 3.66 \text{ revolutions}\]
2. Load $B$ is connected to a double pulley by one of the two inextensible cables shown. The motion of the pulley is controlled by cable $C$, which has a constant acceleration of $9 \text{ cm/s}^2$ and an initial velocity of $12 \text{ cm/s}$, both directed to the right. Determine,

a) The number of revolutions executed by the inner pulley for $t = 2$ seconds.

b) The velocity and change in position of the load $B$ after 2 seconds.

c) The acceleration of point $D$ on the rim of the inner pulley at $t = 0$. 
a) The number of revolutions executed by the inner pulley for $t = 2$ seconds.

\[ v_{C_o} = v_{D_o} = 12 \text{ cm/s} \quad a_{D_t} = 9 \text{ cm/s}^2 \]

\[ r_D = 30 \text{ cm} \]

\[ v_{D_o} = \omega_o r_D \quad \omega_o = \frac{v_{D_o}}{r_D} = \frac{12}{30} = 0.4 \text{ rad/s (cw)} \]

\[ a_{D_t} = \alpha r_D \quad \alpha = \frac{a_{D_t}}{r_D} = \frac{9}{30} = 0.3 \text{ rad/s}^2 \]

\[ \omega = \omega_o + \alpha t \quad \omega = 0.4 + 0.3(2) = 1 \text{ rad/s} \]

\[ \theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2 \quad \theta = 0 + 0.4(2) + \frac{1}{2}0.3(2)^2 = 1.4 \text{ rad (cw)} \]

**Number of revolutions**

\[ 1 \text{ rev} = 2\pi \text{ rad} \]

\[ x \text{ rev} = 1.4 \text{ rad}, \quad x = \frac{1.4}{2\pi} = 0.223 \text{ rev} \]
b) The velocity and change in position of the load $B$ after 2 seconds.

\[ v_B = \omega r_B = 1(50) \text{ cm/s} \]

\[ \Delta y_B = r_B \theta = (50)(1.4) = 70 \text{ cm (upwards)} \uparrow \]

c) The acceleration of point $D$ on the rim of the inner pulley at $t = 0$.

\[ a_{D_t} = 9 \text{ cm/s}^2 \rightarrow \]

at $t = 0$ \quad $\omega_0 = 0.4 \text{ rad/s}$

\[ a_{D_n} = r_D \omega_0^2 = (30)(0.4)^2 = 4.8 \text{ cm/s}^2 \downarrow \]

\[ a_D = \left(a_{D_n}^2 + a_{D_t}^2\right)^{1/2} = 10.2 \text{ cm/s}^2 \]
3. The motor shown is used to turn a wheel by the pulley $A$ attached to it. If the pulley starts rotating from rest with an angular acceleration of $\alpha_A = 2 \text{ rad/s}^2$, determine the magnitudes of the velocity and acceleration of point $P$ on the wheel, after the wheel has turned 10 revolutions. Assume the transmission belt does not slip on the pulley and the wheel.

\[ \theta_B = 20\pi = 62.83 \text{ rad} \quad \omega_A^2 = \omega_O^2 + 2\alpha(\theta_1 - \theta_O) \quad [10 \text{ rev} = 20\pi \text{ rad}] \]

\[ s = \theta_A r_A = \theta_B r_B \quad \omega_A^2 = 0 + 2(2)(167.6) \]

\[ \theta_A 0.15 = 62.83(0.4) \quad \omega_A = 18.31 \text{ rad/s} \]

\[ \theta_A = 167.6 \text{ rad} \]

\[ v_C = \omega_A r_A = \omega_B r_B \quad 18.31(0.15) = \omega_B (0.4) \quad \omega_B = 6.87 \text{ rad/s} \]

\[ v_P = \omega_B r_B = 6.87(0.4) = 2.75 \text{ m/s} \]

\[ a_{Ct} = \alpha_A r_A = \alpha_B r_B \quad \alpha_A r_A = a_{Ct} = a_{Pt} = 0.3 \text{ m/s}^2 \]

\[ \alpha_B = \frac{0.3}{0.4} = 0.75 \text{ rad/s}^2 \]

\[ a_{Pn} = r_B \omega_B^2 = 0.4(6.87)^2 = 18.88 \text{ m/s}^2 \]

\[ a_P = \sqrt{a_{Pn}^2 + a_{Pt}^2} = 18.90 \text{ m/s}^2 \]
In the first approach in rigid body kinematics, the absolute motion analysis, we make use of the geometric relations which define the configuration of the body involved and then proceed to take the time derivatives of the defining geometric relations to obtain velocities and accelerations.

The constrained motion of connected particles is also an absolute motion analysis. For the pulley configurations, the relevant velocities and accelerations were determined by successive differentiation of the lengths of the connecting cables. In rigid body motion, the defining geometric relations include both linear and angular variables and, therefore, the time derivatives of these quantities will involve both linear and angular velocities and linear and angular accelerations.
A wheel of radius $r$ rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center $O$. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.
The wheel rolls to the right from the dashed to the full position without slipping. The linear displacement of the center $O$ is $s$, which is also the arc length $C'A$ along the rim on which the wheel rolls. The radial line $CO$ rotates to the new position $C'O'$ through the angle $\theta$, where $\theta$ is measured from the vertical direction.

If the wheel does not slip, the arc $C'A$ must equal the distance $s$, since in a rigid body all points will have the same displacement. Thus, the displacement relationship and its two time derivatives give

$$s = r \theta \quad (\theta \text{ is in radians})$$

$$\dot{s} = v_o = r \dot{\theta} = rw$$

$$\ddot{s} = a_o = r \ddot{\theta} = r \alpha$$
If the wheel is slowing down, \( a_0 \) will be directed opposite to \( v_0 \) and \( \omega \) and \( \alpha \) will have opposite directions. When point \( C \) has moved along its cycloidal path to \( C' \), its new coordinates and their time derivatives become

\[
\begin{align*}
x &= s - r \sin \theta = r \theta - r \sin \theta = r(\theta - \sin \theta) \\
\dot{x} &= r \dot{\theta} - r \dot{\theta} \cos \theta = r \omega - r \omega \cos \theta = v_0 (1 - \cos \theta) \\
\ddot{x} &= r \ddot{\theta} - r \ddot{\theta} \cos \theta + r \dot{\theta}^2 \sin \theta \\
&= r \alpha - r \alpha \cos \theta + r \omega^2 \sin \theta \\
&= a_0 (1 - \cos \theta) + r \omega^2 \sin \theta
\end{align*}
\]

\[
\begin{align*}
y &= r - r \cos \theta = r(1 - \cos \theta) \\
\dot{y} &= r \dot{\theta} \sin \theta = v_0 \sin \theta \\
\ddot{y} &= r \ddot{\theta} \cos \theta + r \dot{\theta} \sin \theta \\
&= r \omega^2 \cos \theta + r \alpha \sin \theta = r \omega^2 \cos \theta + a_0 \sin \theta
\end{align*}
\]
Thus the acceleration of point C on the rim at the instant of contact with the ground depends only on r and w and is directed toward the center of the wheel. If desired, the velocity and acceleration of C at any position \( \theta \) may be obtained by writing the expressions

\[
\ddot{x} = 0 \quad \ddot{y} = rw^2
\]

Thus the acceleration of point C on the rim at the instant of contact with the ground depends only on r and w and is directed toward the center of the wheel. If desired, the velocity and acceleration of C at any position \( \theta \) may be obtained by writing the expressions

\[
\ddot{v} = \ddot{x} \hat{i} + \ddot{y} \hat{j}
\]

\[
\ddot{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j}
\]

If the wheel slips as it rolls, the foregoing relations will no longer be valid.
The second approach to rigid body kinematics uses the principles of relative motion. In kinematics of particles for motion relative to translating axes, we applied the relative velocity equation

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

to the motions of two particles $A$ and $B$.

We now choose two points on the same rigid body for our two particles. The consequence of this choice is that the motion of one point as seen by an observer translating with the other point must be circular since the radial distance to the observed point from the reference point does not change. This observation is the key to the successful understanding of a large majority of problems in the plane motion of rigid bodies.
The figure shows a rigid body moving in the plane of the figure from position $AB$ to $A'B'$ during time $\Delta t$. This movement may be visualized as occurring in two parts. First, the body translates to the parallel position $A''B'$ with the displacement $\Delta \vec{r}_B$. Second, the body rotates about $B'$ through the angle $\Delta \theta$, from the nonrotating reference axes $x'-y'$ attached to the reference point $B'$, giving rise to the displacement $\Delta \vec{r}_{A/B}$ of $A$ with respect to $B$. 

Motion relative to $B$
To the nonrotating observer attached to $B$, the body appears to undergo fixed axis rotation about $B$ with $A$ executing circular motion.

Point $B$ is arbitrarily chosen as the reference point for attachment of the nonrotating reference axes $x,y$. Point $A$ could have been used just as well, in which case we would observe $B$ to have circular motion about $A$ considered fixed. In this case, the sense of the rotation, counterclockwise direction, is the same whether we choose $A$ or $B$ as the reference, and we see that $\Delta \vec{r}_{B/A} = - \Delta \vec{r}_{A/B}$.
With $B$ as the reference point, the total displacement of $A$ is

\[
\Delta \vec{r}_A = \Delta \vec{r}_B + \Delta \vec{r}_{A/B}
\]

Where $\Delta \vec{r}_{A/B}$ has the magnitude $r\Delta \theta$ as $\Delta \theta$ approaches zero. We note that the relative linear motion $\Delta \vec{r}_{A/B}$ is accompanied by the absolute angular motion $\Delta \theta$, as seen from the translating axes $x'-y'$. Dividing the expression for $\Delta \vec{r}_A$ by the corresponding time interval $\Delta t$ and passing to the limit, we obtain the relative velocity equation

\[
\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}
\]

We should note that in this expression the distance $r$ between $A$ and $B$ remains constant.
The magnitude of the relative velocity is thus seen to be

\[
\nu_{A/B} = \lim_{\Delta t \to 0} \left( \frac{\Delta \vec{r}_{A/B}}{\Delta t} \right) = \lim_{\Delta t \to 0} \left( r \Delta \theta / \Delta t \right)
\]

which, with \( \omega = \dot{\theta} \) becomes

\[
\nu_{A/B} = r \omega
\]

Using \( \vec{r} \) to represent the vector \( \vec{r}_{A/B} \), we may write the relative velocity as the vector

\[
\vec{v}_{A/B} = \vec{\omega} \times \vec{r}
\]

Therefore, the relative velocity equation becomes

\[
\vec{v}_{A} = \vec{v}_{B} + \vec{\omega} \times \vec{r}
\]
Here, $\vec{\omega}$ is the angular velocity vector normal to the plane of the motion in the sense determined by the right hand rule.

It should be noted that the direction of the relative velocity will always be perpendicular to the line joining the points $A$ and $B$.

**Interpretation of the Relative Velocity Equation**

We can better understand the relative velocity equation by visualizing the translation and rotation components separately.
In the figure, point $B$ is chosen as the reference point and the velocity of $A$ is the vector sum of the translational portion $\vec{v}_B$, plus the rotational portion $\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$, which has the magnitude $v_{A/B} = r \omega$, where $|\vec{\omega}| = \dot{\theta}$, the absolute angular velocity of $AB$. The fact that the relative linear velocity is always perpendicular to the line joining the two points $A$ and $B$ is an important key to the solution of many problems.
Solution of the relative velocity equation may be carried out by scalar or vector algebra or by graphic interpretation. In the scalar approach, each term in the relative motion equation may be written in terms of its $\vec{i}$ and $\vec{j}$ components, from which we will obtain two scalar equations.
Relative Acceleration

Consider the equation

\[ \vec{v}_A = \vec{v}_B + \vec{v}_{A/B} \]

which describes the relative velocities of two points \( A \) and \( B \) in plane motion in terms of nonrotating reference axes. By differentiating the equation with respect to time, we obtain the relative acceleration equation, which is

\[ \dot{\vec{v}}_A = \dot{\vec{v}}_B + \dot{\vec{v}}_{A/B} \quad \text{or} \quad \]

\[ \vec{a}_A = \vec{a}_B + \vec{a}_{A/B} \]

This equation states that the acceleration of point \( A \) equals the vector sum of the acceleration of point \( B \) and the acceleration which \( A \) appears to have to a nonrotating observer moving with \( B \).
Relative Acceleration Due to Rotation

If points $A$ and $B$ are located on the same rigid body and in the plane of motion, the distance $r$ between them remains constant so that the observer moving with $B$ perceives $A$ to have circular motion about $B$. Because the relative motion is circular, it follows that the relative acceleration term will have both a normal component directed from $A$ toward $B$ due to the change of direction of $\vec{v}_{A/B}$ and a tangential component perpendicular to $AB$ due to the change in magnitude of $\vec{v}_{A/B}$. Thus, we may write,

$$\vec{a}_A = \vec{a}_B + (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_t$$

Where the magnitudes of the relative acceleration components are

$$\left(\vec{a}_{A/B}\right)_n = \frac{v_{A/B}^2}{r} = r\omega^2$$

$$\left(\vec{a}_{A/B}\right)_t = \dot{v}_{A/B} = r\alpha$$
In vector notation the acceleration components are

\[
\left( \vec{a}_{A/B} \right)_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})
\]
\[
\left( \vec{a}_{A/B} \right)_t = \vec{\alpha} \times \vec{r}
\]

In these relationships, \( \vec{\omega} \) is the angular velocity and \( \vec{\alpha} \) is the angular acceleration of the body. The vector locating \( A \) from \( B \) is \( \vec{r} \). It is important to observe that the relative acceleration terms depend on the respective absolute angular velocity and the absolute angular acceleration.

The relative acceleration equation, thus, becomes

\[
\vec{a}_A = \vec{a}_B + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r}
\]
Interpretation of the Relative Acceleration Equation

The meaning of relative acceleration equation is indicated in the figure which shows a rigid body in plane motion with points $A$ and $B$ moving along separate curved paths with absolute accelerations $\vec{a}_A$ and $\vec{a}_B$.

Contrary to the case with velocities, the accelerations $\vec{a}_A$ and $\vec{a}_B$ are, in general, not tangent to the paths described by $A$ and $B$ when these paths are curvilinear.
The figure shows the acceleration of $A$ to be composed of two parts: the acceleration of $B$ and the acceleration of $A$ with respect to $B$. A sketch showing the reference point as fixed is useful in disclosing the correct sense of the two components of the relative acceleration term.
Alternatively, we may express the acceleration of $B$ in terms of the acceleration of $A$, which puts the nonrotating reference axes on $A$ rather than $B$. This order gives

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Here $\vec{a}_{B/A}$ and its $n$ and $t$ components are the negatives of $\vec{a}_{A/B}$ and its $n$ and $t$ components ($\vec{a}_{A/B} = -\vec{a}_{B/A}$).
Solution of the Relative Acceleration Equation

As in the case of the relative velocity equation, the relative acceleration equation may be carried out by scalar or vector algebra or by graphical construction.

Because the normal acceleration components depend on velocities, it is generally necessary to solve for the velocities before the acceleration calculations can be made.