1. A cable stretched between the fixed supports A and B is under a tension $T$ of 900 N. Express the tension as a vector using the unit vectors $\vec{i}$ and $\vec{j}$, first, as a force $\vec{T_A}$ acting on A and second, as a force $\vec{T_B}$ acting on B.
2. Determine the $x$-$y$ components of the tension $T$ which is applied to point $A$ of the bar $OA$. Neglect the effects of the small pulley at $B$. Assume that $r$ and $\theta$ are known. Also determine the $n$-$t$ components of the tension $T$ for $T=100$ N and $\theta=35^\circ$. 
\[AB = \sqrt{(r - r \sin \theta)^2 + (r + r \cos \theta)^2} = r \sqrt{3 + 2 \cos \theta - 2 \sin \theta}\]

\[
\cos \beta = \frac{r + r \cos \theta}{r \sqrt{3 + 2 \cos \theta - 2 \sin \theta}} = \frac{1 + \cos \theta}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}
\]

\[
\sin \beta = \frac{r - r \sin \theta}{r \sqrt{3 + 2 \cos \theta - 2 \sin \theta}} = \frac{1 - \sin \theta}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}
\]

\[
T_x = T \cos \beta = T \frac{1 + \cos \theta}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}
\]

\[
T_y = -T \sin \beta = T \frac{\sin \theta - 1}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}
\]

**n-t coordinates** (for \(\theta = 35^\circ\) and \(T = 100\) N)

\[
\beta = \arctan \left( \frac{1 - \sin 35}{1 + \cos 35} \right) = 13.19^\circ
\]

\[
T_n = T \cos(\theta + \beta) = 100 \cos(35 + 13.19) = 66.67\ N
\]

\[
T_t = T \sin(\theta + \beta) = 100 \sin(35 + 13.19) = 74.54\ N
\]
In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90 N force $P$ on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts on the robot along axes (a) parallel and perpendicular to the arm $AB$, and (b) parallel and perpendicular to the arm $BC$. 

![Diagram of robot arm and cylinder]
4. The unstretched length of the spring is \( r \). When pin \( P \) is in an arbitrary position \( \theta \), determine the \( x \)- and \( y \)-components of the force which the spring exerts on the pin. Evaluate your answer for \( r=400 \) mm, \( k=1.4 \) kN/m and \( \theta=40^\circ \).
5. Three forces act on the bracket. Determine the magnitude and direction $\theta$ of $F_2$ so that the resultant force is directed along the positive $u$ axis and has a magnitude of 50 N.

$F_1 = 80\, \text{N}$

$F_3 = 52\, \text{N}$

$F_2$
if \( R = 50 \text{ N} \) \( \theta = \? \) \( F_2 = \? \)

**Resultant**

\[
\vec{R} = R \cos 25^\circ \hat{i} - R \sin 25^\circ \hat{j} \quad \Rightarrow \quad \vec{R} = 50 \cos 25^\circ \hat{i} - 50 \sin 25^\circ \hat{j}
\]

\[
\vec{R} = 45.315 \hat{i} - 21.13 \hat{j}
\]

\[
\vec{F}_1 = 80 \hat{i}
\]

\[
\vec{F}_2 = F_2 \cos(\theta + 25^\circ) \hat{i} - F_2 \sin(\theta + 25^\circ) \hat{j}
\]

\[
\vec{F}_3 = 52 \left( \frac{5}{13} \hat{i} + \frac{12}{13} \hat{j} \right) = 20 \hat{i} + 48 \hat{j}
\]

\[
\sum \vec{F} = \vec{R} = \sum F_x \hat{i} + \sum F_y \hat{j}
\]

\[
\sum F_x = 80 + F_2 \cos(\theta + 25^\circ) + 20 = 45.315
\]

\[
F_2 \cos(\theta + 25^\circ) = -54.685 \quad \text{①}
\]

\[
\sum F_y = -F_2 \sin(\theta + 25^\circ) + 48 = -21.13
\]

\[
F_2 \sin(\theta + 25^\circ) = 69.13 \quad \text{②}
\]

\[
\frac{F_2 \sin(\theta + 25^\circ)}{F_2 \cos(\theta + 25^\circ)} = \frac{69.13}{-54.685}
\]

\[
\tan(\theta + 25^\circ) = -1.264
\]

\[
\theta = 103.35^\circ \quad F_2 = 88.14 \text{ N}
\]
6. The turnbuckle $T$ is tightened until the tension in cable $OA$ is 5 kN. Express the force $\vec{F}$ acting on point $O$ as a vector. Determine the projection of $\vec{F}$ onto the $y$-axis and onto line $OB$. Note that $OB$ and $OC$ lies in the $x$-$y$ plane.
7. The cable $BC$ carries a tension of 750 N. Write this tension as a force $\vec{T}$ acting on point $B$ in terms of the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$. The elbow at $A$ forms a right angle.
8. In opening a door which is equipped with a heavy-duty return mechanism, a person exerts a force $P$ of magnitude 40 N as shown. Force $P$ and the normal $n$ to the face of the door lie in a vertical plane. Express $P$ as a vector and determine the angles $\theta_x$, $\theta_y$ and $\theta_z$ which the line of action of $P$ makes with the positive $x$-, $y$- and $z$-axes.
9. The spring of constant $k = 2.6 \text{ kN/m}$ is attached to the disk at point A and to the end fitting at point B as shown. The spring is unstretched when $\theta_A$ and $\theta_B$ are both zero. If the disk is rotated $15^\circ$ clockwise and the end fitting is rotated $30^\circ$ counterclockwise, determine a vector expression for the force which the spring exerts at point A.

Problems (Force Systems)
10 An overhead crane is used to reposition the boxcar within a railroad car-repair shop. If the boxcar begins to move along the rails when the $x$-component of the cable tension reaches 3 kN, calculate the necessary tension $T$ in the cable. Determine the angle $\theta_{xy}$ between the cable and the vertical $x$-$y$ plane.
T_x = 3 kN, calculate tension \( T \), the angle \( \theta_{xy} \) between the cable and the vertical \( x-y \) plane.

**Unit vector of \( \vec{T} \)**

\[
\vec{n}_T = \frac{5\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{5^2 + 4^2 + 1^2}}
\]

\[
\vec{n}_T = 0.77\hat{i} + 0.617\hat{j} + 0.154\hat{k}
\]

**\( \vec{T} \) in vector form**

\[
\vec{T} = T (0.77\hat{i} + 0.617\hat{j} + 0.154\hat{k})
\]

**x-component of \( \vec{T} \)**

\[0.77T = 3 \quad \Rightarrow \quad T = 3.896 \text{ kN} \quad \text{(magnitude of \( \vec{T} \))}
\]

**\( y \) and \( z \)-components of \( \vec{T} \)**

\[
T_y = 0.617(3.896) = 2.4 \text{ kN} \quad \quad T_z = 0.154(3.896) = 0.6 \text{ kN}
\]

\[
\vec{T} = 3\hat{i} + 2.4\hat{j} + 0.6\hat{k}
\]

\[
T_{xy} = \sqrt{T_x^2 + T_y^2} = \sqrt{3^2 + 2.4^2} = 3.84 \text{ kN}
\]

\[
\cos \theta_{xy} = \frac{T_{xy}}{T} = \frac{3.84}{3.89} = 0.896 \quad \Rightarrow \quad \theta_{xy} = 9.598^\circ
\]
11. The rectangular plate is supported by hinges along its side $BC$ and by the cable $AE$. If the cable tension is 300 N, determine the projection onto line $BC$ of the force exerted on the plate by the cable. Note that $E$ is the midpoint of the horizontal upper edge of the structural support.
If $T=300$ N, determine the projection onto line $BC$ of the force exerted on the plate by the cable.

**Coordinates of points $A$, $B$, $C$ and $E$ with respect to coordinate system**

$A$ (400, 0, 0)  $B$ (0, 0, 0)

$C$ (0, 1200sin25, $-1200cos25$)

$C$ (0, 507.14, $-1087.57$)

$E$ (0, 1200sin25, $-600cos25$)

$E$ (0, 507.14, $-543.78$)

$$\vec{T} = T\vec{n}_T = 300\left(-\frac{400i + 507.14j - 543.78k}{844.33}\right)$$

$$\vec{T} = -142.12i + 180.19j - 193.21k$$

**Unit vector of line $BC$**

$$\vec{n}_{BC} = -\cos 25k + \sin 25j = 0.423j - 0.906k$$

**Projection of $T$ onto line $BC$**

$$T_{BC} = \vec{T} \cdot \vec{n}_{BC} = \left(-142.12i + 180.19j - 193.21k\right) \cdot \left(0.423j - 0.906k\right)$$

$$T_{BC} = 251.26 \text{ N}$$
12. The y and z scalar components of a force are 100 N and 200 N, respectively. If the direction cosine \( l = \cos \theta_x \) of the line of action of the force is –0.5, write \( \vec{F} \) as a vector.

\[
F_y = 100 \text{ N} \quad F_z = 200 \text{ N} \quad l = -0.5
\]

\[
l^2 + m^2 + n^2 = 1 \quad \Rightarrow \quad m^2 + n^2 = 1 - 0.5^2 \quad \Rightarrow \quad m^2 + n^2 = 0.75
\]

\[
\sqrt{F_y^2 + F_z^2} = 223.61 \text{ N} \quad (= F_{yz})
\]

\[
F_y = F \cos \theta_y = Fm \quad F_z = F \cos \theta_z = Fn
\]

\[
F \sqrt{0.75} = 223.61 \quad \Rightarrow \quad F = 258.2 \text{ N}
\]

\[
F_x = F \cos \theta_x = -129.1 \text{ N}
\]

\[
\vec{F} = -129.1 \hat{i} + 100 \hat{j} + 200 \hat{k}
\]
13. Determine the parallel and normal components of force $\mathbf{F}$ in vector form with respect to a line passing through points $A$ and $B$.

**Cartesian components of $\mathbf{F}$**

\[ F_{xy} = 75 \frac{5}{\sqrt{5^2 + 3^2}} = 64.311 \text{ kN} \]

\[ F_x = F_{xy} \cos 50 = 41.34 \text{ kN} \]

\[ F_y = F_{xy} \sin 50 = 49.265 \text{ kN} \]

\[ F_z = 75 \frac{3}{\sqrt{5^2 + 3^2}} = 38.59 \text{ kN} \]

\[ \mathbf{F} = 41.34 \mathbf{i} + 49.265 \mathbf{j} + 38.59 \mathbf{k} \]
\[ \vec{F} = 41.34\hat{i} + 49.265\hat{j} + 38.59\hat{k} \]

**Unit vector of line AB**

\[ \vec{n}_{AB} = \frac{3\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{3^2 + 2^2 + 3^2}} = 0.639\hat{i} + 0.426\hat{j} + 0.639\hat{k} \]

**Parallel component of \( \vec{F} \) to line AB (its scalar value):**

\[ F_\parallel = \vec{F} \cdot \vec{n}_{AB} \]

\[ F_\parallel = \left(41.34\hat{i} + 49.265\hat{j} + 38.59\hat{k}\right) \cdot \left(0.639\hat{i} + 0.426\hat{j} + 0.639\hat{k}\right) \]

\[ F_\parallel = (41.34)(0.639) + (49.265)(0.426) + (38.59)(0.639) = 72.14 \text{ kN} \]

**Parallel component of \( \vec{F} \) to line AB (in vector form):**

\[ \vec{F}_\parallel = F_\parallel \vec{n}_{AB} = 72.14\left(0.639\hat{i} + 0.426\hat{j} + 0.639\hat{k}\right) = 46.14\hat{i} + 30.76\hat{j} + 46.14\hat{k} \]

**Normal component of \( \vec{F} \) to line AB (in vector form):**

\[ \vec{F}_\perp = \vec{F} - \vec{F}_\parallel = -4.8\hat{i} + 18.505\hat{j} - 7.55\hat{k} \]
14. Determine the magnitude and direction angles of the resultant force acting on the bracket.

**Resultant**

\[ \vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 \]

\[ \vec{F}_1 = -450 \cos 45 \sin 30 \vec{i} + 450 \cos 45 \cos 30 \vec{j} + 450 \sin 45 \vec{k} \]

\[ \vec{F}_1 = -159.1 \vec{i} + 275.57 \vec{j} + 318.2 \vec{k} \]
**Direction angles for** $\vec{F}_2$

\[ \theta_x = 45^\circ \quad \theta_y = 60^\circ \quad \theta_z > 90^\circ \]

**Direction cosines**

\[
\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1
\]

\[
\left\{ l^2 + m^2 + n^2 = 1 \right\}
\]

\[
\cos^2 45 + \cos^2 60 + \cos^2 \theta_z = 1
\]

\[
\cos^2 \theta_z = 0.25 \implies \cos \theta_z = \pm 0.5
\]

\[
\theta_z > 90^\circ \implies \cos \theta_z = -0.5 \implies \theta_z = 120^\circ
\]

\[
\vec{F}_2 = 600 \cos 45 \hat{i} + 600 \cos 60 \hat{j} + 600 \cos 120 \hat{k}
\]

\[
\vec{F}_2 = 424.26 \hat{i} + 300 \hat{j} - 300 \hat{k}
\]
\[ \vec{F}_1 = -159.1 \hat{i} + 275.57 \hat{j} + 318.2 \hat{k} \quad \vec{F}_2 = 424.26 \hat{i} + 300 \hat{j} - 300 \hat{k} \]

**Resultant**  
\[ \vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 \]
\[ \vec{R} = (-159.1 + 424.26) \hat{i} + (275.57 + 300) \hat{j} + (318.2 - 300) \hat{k} \]
\[ \vec{R} = 265.16 \hat{i} + 575.57 \hat{j} + 18.2 \hat{k} \]

**Magnitude of Resultant Force**  
\[ |\vec{R}| = R = \sqrt{265.16^2 + 575.57^2 + 18.2^2} = 633.97 \text{ N} \]

**Direction Cosines of Resultant Force**
\[ \cos \theta_x = \frac{R_x}{R} \quad \cos \theta_y = \frac{R_y}{R} \quad \cos \theta_z = \frac{R_z}{R} \]
\[ \cos \theta_x = \frac{265.16}{633.97} = 0.418 \quad \cos \theta_y = \frac{575.57}{633.97} = 0.907 \quad \cos \theta_z = \frac{18.2}{633.97} = 0.029 \]

**Direction angles for \( \vec{R} \)**
\[ \theta_x = \arccos(0.418) = 65.3^\circ \quad \theta_y = \arccos(0.907) = 24.9^\circ \quad \theta_z = \arccos(0.029) = 88.3^\circ \]
15. Express the force $\vec{F}$ as a vector in terms of unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$. Determine the direction angles $\theta_x$, $\theta_y$ and $\theta_z$ which $\vec{F}$ makes with the positive $x$-, $y$-, and $z$-axes.
**Position vector**

\[ \vec{r}_{B/A} = \overrightarrow{AB} = (-25 - 15)i + [50 - (-20)]j + [40 - (-25)]k \]

\[ \vec{r}_{B/A} = \overrightarrow{AB} = -40i + 70j + 65k \]

**Unit vector**

\[ \vec{n} = \frac{\vec{r}_{B/A}}{|\vec{r}_{B/A}|} = \frac{-40i + 70j + 65k}{\sqrt{40^2 + 70^2 + 65^2}} \]

\[ \vec{n} = -0.386i + 0.676j + 0.627k \]

\[ \vec{F} = F\vec{n} = 750\left(-0.386i + 0.676j + 0.627k\right) = -289.5i + 507j + 470.25k \]

**Direction cosines**

\[ l = -0.386 \quad m = 0.676 \quad n = 0.627 \quad \{l^2 + m^2 + n^2 = 1\} \]

\[ \begin{cases} l = \cos \theta_x = -0.386 \quad \Rightarrow \quad \theta_x = 112.7^\circ \\ m = \cos \theta_y = 0.676 \quad \Rightarrow \quad \theta_y = 47.47^\circ \\ n = \cos \theta_z = 0.627 \quad \Rightarrow \quad \theta_z = 51.17^\circ \end{cases} \]