PROBLEMS ON
THREE DIMENSIONAL
EQUILIBRIUM OF
RIGID BODIES
1. The pole is subjected to the two forces shown. Determine the components of reaction of A assuming it to be a ball-and-socket joint.
The shaft assembly (consisting of welded pieces $AB$, $ED$ and $CD$) is supported by a thrust bearing at $A$ and a radial bearing at $B$. The assembly is subjected to a force $\vec{F}$ at $C$ and a couple $\vec{M}$. If it is known that the $y$ component of the reaction at bearing $B$ is $104 \, \hat{j}$ (N), determine the vector expressions of the force $\vec{F}$, couple $\vec{M}$ and the bearing reactions at $A$ and $B$. Link $ED$ lies in the $yz$ plane. $ED=250$ mm.
\[ \theta = \arctan \left( \frac{3}{4} \right) \]

\[ \overline{DE} = 250 \text{ mm} \]

\[ \text{400 mm} \]

\[ \text{350 mm} \]

\[ \text{120 mm} \]
3. Structure $ABCD$ is supported by a collar at $D$ that can rotate and slide along bar $EF$ which is fixed and is frictionless. Structure $ABCD$ makes contact with smooth surfaces at $A$ and $C$ where normal direction $\vec{n}$ to the surface at $A$ lies in a plane that is parallel to the $xy$ plane. Force $P$ is parallel to the y axis. If $P=10$ kN, determine the reactions at $A$, $C$ and $D$. Note that collar $D$ acts like a wide radial bearing.
4. Because of friction a very large force $T$ is required to raise the 10 kg body when the cable is wound around a fixed rough cylinder as shown. For the static equilibrium values shown, determine all of the reactions at point O. The 0.3 m dimension refers to the point A where the cable loses contact with the cylinder. Neglect the weight of the fixed cylinder.
\[ W = 10 \times (9.81) \ N = 0.098 \ kN \]
Reactions at O:

\[ \vec{R}_o = O_x \vec{i} + O_y \vec{j} + O_z \vec{k} \]

\[ \vec{M}_o = M_x \vec{i} + M_y \vec{j} + M_z \vec{k} \]

\[ \vec{T} = T \sin 30 \vec{i} - T \cos 30 \cos 20 \vec{j} - T \cos 30 \sin 20 \vec{k} \]

\[ \vec{T} = 15 \vec{i} - 24.41 \vec{j} - 8.89 \vec{k} \]

\[ \vec{W} = -10(9.81) \vec{i} = -98.1 \vec{i} \ [N] = -0.098 \vec{i} \ [kN] \]
\[ \sum M_O = 0 \]
\[ (1100 \bar{k} + 30 \bar{j}) \times (-0.098 \bar{i}) + (26 \bar{i} + 15 \bar{j} + 800 \bar{k}) \times (15 \bar{i} - 24.41 \bar{j} - 8.89 \bar{k}) \]
\[ + M_x \bar{i} + M_y \bar{j} + M_z \bar{k} = 0 \]
\[ M_x = -19394.65 \text{ kN} \cdot \text{mm} \quad M_y = -12123.34 \text{ kN} \cdot \text{mm} \quad M_z = 856.72 \text{ kN} \cdot \text{mm} \]

\[ \Sigma F_x = 0 \quad O_x + T_x - 0.098 = 0 \quad \Rightarrow \quad O_x = -14.902 \text{ kN} \]
\[ \Sigma F_y = 0 \quad O_y - T_y = 0 \quad \Rightarrow \quad O_y = 24.41 \text{ kN} \]
\[ \Sigma F_z = 0 \quad O_z - T_z = 0 \quad \Rightarrow \quad O_z = 8.89 \text{ kN} \]
5. The uniform panel door has a mass of 30 kg and is prevented from opening by the strut C, which is a light two-force member whose upper end is secured under the door knob and whose lower end is attached to a rubber cup which does not slip on the floor. Of the door hinges A and B, only B can support force in the vertical z-direction. Calculate the compression C in the strut and the horizontal components of the forces supported by hinges A and B when a horizontal force \( P = 200 \text{ N} \) is applied normal to the plane of the door as shown.
\( \mathbf{O} (0, 0, 0) \quad \mathbf{A} (-1.2, 0, 1.8) \quad \mathbf{B} (-1.2, 0, 0.2) \\
\mathbf{G} (-0.6, 0, 1) \quad \mathbf{E} (0, 0, 2) \quad \mathbf{D} (-0.2, 0, 1) \\
\mathbf{H} (-0.2, 0.75, 0) \\

\begin{align*}
\mathbf{\bar{n}}_{DH} &= \frac{-0.75 \mathbf{j} + \mathbf{k}}{1.25} = -0.6 \mathbf{j} + 0.8 \mathbf{k} \\
\mathbf{\bar{F}}_C &= \mathbf{F}_C \mathbf{\bar{n}}_{DH} = \mathbf{F}_C (-0.6 \mathbf{j} + 0.8 \mathbf{k}) \\
\mathbf{\bar{A}} &= A_x \mathbf{i} + A_y \mathbf{j} \\
\mathbf{\bar{B}} &= B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \\
\mathbf{\bar{W}} &= -30(9.81) \mathbf{k} = -294.3 \mathbf{k} \quad [N] \\
\mathbf{\bar{P}} &= 200 \mathbf{j} \quad [N] \\

\sum M_B &= 0 \\
\sum M_B &= \left[(0.6 \mathbf{i} + 0.8 \mathbf{k}) \times (-294.3 \mathbf{k})\right] + \left[(\mathbf{i} + 0.8 \mathbf{k}) \times \mathbf{F}_C (-0.6 \mathbf{j} + 0.8 \mathbf{k})\right] \\
&+ \left[(1.2 \mathbf{i} + 1.8 \mathbf{k}) \times (200 \mathbf{j})\right] + \left[(1.6 \mathbf{k}) \times (A_x \mathbf{i} + A_y \mathbf{j})\right]
\end{align*}
\[ \Sigma \vec{M}_B = (0.48F_C - 1.6A_y - 360)\hat{i} + (-0.8F_C + 1.6A_x + 176.58)\hat{j} + (-0.6F_C + 240)\hat{k} = 0 \]

\[ \hat{k} \quad -0.6F_C + 240 = 0 \quad \Rightarrow \quad F_C = 400 \text{ N} \]

\[ \hat{i} \quad 0.48F_C - 360 = 1.6A_y \quad \Rightarrow \quad A_y = -105 \text{ N} \]

\[ \hat{j} \quad +1.6A_x = 0.8F_C - 176.58 \quad \Rightarrow \quad A_x = 89.64 \text{ N} \]

\[ A_{xy} = 138.06 \text{ N} \]

\[ \Sigma F_x = 0 \quad A_x + B_x = 0 \quad B_x = -A_x = -89.64 \text{ N} \]

\[ \Sigma F_y = 0 \quad A_y + B_y + 200 - 0.6F_C = 0 \quad B_y = 145 \text{ N} \]

\[ B_{xy} = 170.47 \text{ N} \]
6. The control surface of an aircraft is supported by a thrust bearing at point $C$ and is actuated by a bar connected to point $A$. The 1 kN force acts in the negative $z$ direction, and the line connecting points $A$ and $B$ is parallel to the $z$ axis. Determine the value of force $F$ needed for equilibrium and all support reactions.
Reactions at C:

\[ \vec{R}_C = C_x \vec{i} + C_y \vec{j} + C_z \vec{k} \quad , \quad \vec{M}_C = M_x \vec{i} + M_z \vec{k} \]

\[ \vec{F} = \frac{14}{15} F \vec{i} + \frac{2}{15} F \vec{j} - \frac{5}{15} F \vec{k} \quad , \quad \vec{F}_D = -1 \vec{k} \]

\[ \sum \vec{M}_C = 0 \]

\[ \left( -0.2 \vec{j} - 0.1 \vec{k} \right) \times F \left( \frac{14}{15} \vec{i} + \frac{2}{15} \vec{j} - \frac{5}{15} \vec{k} \right) + \left( 0.3 \vec{i} + 0.6 \vec{j} \right) \times \left( - \vec{k} \right) + M_x \vec{i} + M_z \vec{k} = 0 \]

\[ \frac{2.8}{15} F \vec{k} + \frac{1}{15} F \vec{i} - \frac{1.4}{15} F \vec{j} + \frac{0.2}{15} F \vec{i} + 0.3 \vec{j} - 0.6 \vec{i} + M_x \vec{i} + M_z \vec{k} = 0 \]

\[ \vec{i} \quad \Rightarrow \quad \frac{1}{15} F + \frac{0.2}{15} F - 0.6 + M_x = 0 \]

\[ \vec{j} \quad \Rightarrow \quad -\frac{1.4}{15} F + 0.3 = 0 \quad \Rightarrow \quad F = 3.21 \text{ kN} \quad M_x = 0.343 \text{ kN} \cdot \text{m} \]

\[ \vec{k} \quad \Rightarrow \quad \frac{2.8}{15} F + M_z = 0 \quad \Rightarrow \quad M_z = -0.6 \text{ kN} \]
Reactions at C:

\[ \vec{R}_C = C_x \vec{i} + C_y \vec{j} + C_z \vec{k}, \quad \vec{M}_C = M_x \vec{i} + M_z \vec{k} \]

\[ \vec{F} = \frac{14}{15} \vec{F}_i + \frac{2}{15} \vec{F}_j - \frac{5}{15} \vec{F}_k, \quad \vec{F}_D = -1\vec{k} \]

\[ \Sigma F_x = 0 \quad \frac{14}{15} F + C_x = 0 \quad \Rightarrow \quad C_x = -2.996 \text{ kN} \]

\[ \Sigma F_y = 0 \quad \frac{2}{15} F + C_y = 0 \quad \Rightarrow \quad C_y = -0.428 \text{ kN} \]

\[ \Sigma F_z = 0 \quad -\frac{5}{15} F + C_z - 1 = 0 \quad \Rightarrow \quad C_z = 2.07 \text{ kN} \]
7. The shaft, lever and handle are welded together and constitute a single rigid body. Their combined mass is 28 kg with mass center at $G$. The assembly is mounted in bearings $A$ and $B$, and rotation is prevented by link $CD$. Determine the forces exerted on the shaft by bearings $A$ and $B$ while the 30 N·m couple is applied to the handle as shown.
\[ \vec{M}_A = 0 \]
\[ (0.6 \vec{j} + 0.1 \vec{k}) \times F (0.6 \vec{i} - 0.8 \vec{j}) + (0.3 \vec{k} + 0.22 \vec{j}) \times (-28 \cdot 9.81 \vec{i}) + 30 \vec{k} + (0.6 \vec{k}) \times (B_x \vec{i} + B_y \vec{j}) = 0 \]
\[ 0.06F \vec{j} + 0.08F \vec{i} - 0.36 \vec{F} \vec{k} - 82.4 \vec{j} + 60.43 \vec{k} + 30 \vec{k} + 0.6B_x \vec{j} - 0.6B_y \vec{i} = 0 \]

\[ \vec{F} = 0.6\vec{F} \vec{i} - 0.8\vec{F} \vec{j} \]

\[ \begin{align*}
\sum M_A &= 0 \\
(0.6 \vec{j} + 0.1 \vec{k}) \times F (0.6 \vec{i} - 0.8 \vec{j}) + (0.3 \vec{k} + 0.22 \vec{j}) \times (-28 \cdot 9.81 \vec{i}) + 30 \vec{k} + (0.6 \vec{k}) \times (B_x \vec{i} + B_y \vec{j}) &= 0 \\
0.06F \vec{j} + 0.08F \vec{i} - 0.36F \vec{k} - 82.4 \vec{j} + 60.43 \vec{k} + 30 \vec{k} + 0.6B_x \vec{j} - 0.6B_y \vec{i} &= 0
\end{align*} \]

\[ \begin{align*}
\vec{i} & \Rightarrow -0.36F + 90.43 = 0 \quad \Rightarrow \quad F = 251.2 \ N \\
\vec{j} & \Rightarrow 0.08F - 0.6B_y = 0 \quad \Rightarrow \quad B_y = 33.49 \ N \\
\vec{k} & \Rightarrow 0.06F - 82.4 + 0.6B_x = 0 \quad B_x = 112.2 \ N
\end{align*} \]
\[ F = 0.6 \vec{F}_i - 0.8 \vec{F}_j \]

\[ F = 251.2 \text{ N} \]
\[ B_y = 33.49 \text{ N} \]
\[ B_x = 112.2 \text{ N} \]

\[ \sum F_x = 0 \implies 0.6F + A_x + 112.2 - 274.7 = 0 \]
\[ A_x = 11.78 \text{ N} \]

\[ \sum F_y = 0 \implies -0.8F + A_y + B_y = 0 \]
\[ A_y = 167.46 \text{ N} \]
8. The electric sander has a mass of 3 kg with mass center at $G$ and is held in a slightly tilted position ($z$-axis vertical) so that the sanding disk makes contact at its top with the surface being sanded. The sander is gripped by its handles at $B$ and $C$. If the normal force $R$ against the disk is maintained at 20 N and is due entirely to the force component $B_x$ (i.e., $C_x = 0$), and if the friction force $F$ acting on the disk is 60 percent of $R$, determine the components of the couple $M$ which must be applied to the handle at $C$ to hold the sander in position. Assume that half of the weight is supported at $C$. 

![Diagram of the electric sander with labeled forces and coordinates.]
\[ R = 20 \text{ N} \quad \Rightarrow \quad F = 20 \cdot \frac{60}{100} = 12 \text{ N} \]

\[ C_x = 0 \quad C_z = \frac{W}{2} = \frac{3 \cdot (9.81)}{2} = 14.715 \text{ N} \]

\[ \sum F_x = 0 \quad \Rightarrow \quad 20 - B_x = 0 \quad B_x = 20 \text{ N} \]

\[ \sum F_y = 0 \quad \Rightarrow \quad 12 - C_y = 0 \quad C_y = 12 \text{ N} \]

\[ \sum F_z = 0 \quad \Rightarrow \quad W - B_z - C_z = 0 \quad B_z = C_z = 14.715 \text{ N} \]

\[ \sum \vec{M}_C = 0 \]
\[
(-0.12\vec{i} - 0.4\vec{k}) \times (20\vec{i} + 12\vec{j})
+ (-0.04\vec{i} + 0.2\vec{j} - 0.3\vec{k}) \times (-20\vec{i} - 14.715\vec{k})
+ (-0.04\vec{i} - 0.3\vec{k}) \times (29.43\vec{k})
+ M_x\vec{i} + M_y\vec{j} + M_z\vec{k} = 0
\]

\[ M_x = -1.857 \text{ Nm} \quad M_y = 1.411 \text{ Nm} \quad M_z = -2.56 \text{ Nm} \]