PLANE KINETICS OF RIGID BODIES (PROBLEMS)
1. In the mechanism shown, the flywheel has a mass of 50 kg and radius of gyration about its center of 160 mm. Uniform connecting rod AB has a mass of 10 kg. Mass of the piston B is 15 kg. Flywheel is rotating by the couple $T$ ccw at a constant rate 50 rad/s. When $\theta=53^\circ$ determine the angular velocity and angular acceleration of the connecting rod AB ($\omega_{AB}$ ve $\alpha_{AB}$). What are the forces transmitted by the pins at A and B? Neglect the friction. Take $\sin 53=0.8$, $\cos 53=0.6$. 

![Diagram of mechanism with flywheel, connecting rod AB, and piston B with angle $\theta$.]
2. Crank $\mathbf{AB}$ rotates with an angular velocity of $\omega_{AB} = 6 \text{ rad/s}$ and angular acceleration of $\alpha_{AB}=2 \text{ rad/s}^2$, both in counterclockwise direction. Roller $\mathbf{C}$ can slide along the circular slot within the fixed plate. For the position shown, angular velocity and angular acceleration of rod $\mathbf{BC}$ are $\omega_{BC} =2.9 \text{ rad/s}$ (counterclockwise) and $\alpha_{BC}=37.5 \text{ rad/s}^2$ (counterclockwise). The masses of uniform bars $\mathbf{AB}$ and $\mathbf{BC}$ are $m_{AB}=2 \text{ kg}$ and $m_{BC}=5 \text{ kg}$. Mass of the roller $\mathbf{C}$ and friction can be neglected. Determine the reactions supported by the pins $\mathbf{B}$ and $\mathbf{C}$?
3. Member \( AO \) is rotating at a **constant** angular velocity of \( \omega_{AO} = 5 \text{ rad/s} \) in ccw direction by a torque \( T = 10 \text{ N\cdotm} \). The mass of the uniform slender bar \( AO \) is 2 kg. Bar \( AB \) has a mass of 5 kg and a radius of gyration with respect to its mass center \( G \) of 400 mm. For the position shown, angular velocity and angular acceleration of rod \( BC \) are \( \omega_{AB} = 1.19 \text{ rad/s} \) (counterclockwise) and \( \alpha_{AB} = 11.79 \text{ rad/s}^2 \) (counterclockwise). Gear \( D \) can be assumed as a uniform thin disk with the radius of \( R = 0.3 \text{ m} \). Its mass is 8 kg and it rotates with an angular velocity of 7.93 rad/s in counterclockwise direction and an angular acceleration of 5.67 rad/s\(^2\) in clockwise direction. Determine the reactions supported by the pins \( A \) and \( B \) and contact point \( C \).
4. The masses of uniform bars $AB$ and $BC$ are $m_{AB} = 2$ kg and $m_{BC} = 1$ kg, respectively. Bar $BC$ is pin connected to a fixed support at $C$. Bar $AB$ is pin connected at $A$ to a uniform wheel of radius $R = 0.50$ m and mass $m_w = 5$ kg. At the instant shown, $A$ is vertically aligned with $O$, bar $AB$ is horizontal and bar $BC$ is vertical. For the given instant, bar $BC$ is rotating with an angular velocity of $2$ rad/s and an angular acceleration of $1.2$ rad/s$^2$, both in clockwise direction. Assuming that the wheel rolls without slipping, determine the force $P$ that is applied to the wheel. Also determine the coefficient of friction between the wheel and the surface.
**KINEMATIC ANALYSIS: VELOCITY**

**MEMBER BC** 
\[ \vec{v}_B = \vec{v}_C + \vec{v}_{BC} = \vec{\omega}_{BC} \times \vec{r}_{BC} = -2\vec{k} \times (0.95\vec{j}) = 1.9\vec{i} \]

**MEMBER AB** 
\[ \vec{v}_A = \vec{v}_B + \vec{v}_{AB} = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{AB} = 1.9\vec{i} + \omega_{AB}\vec{k} \times (-1.25\vec{i}) = 1.9\vec{i} - 1.25\omega_{AB}\vec{j} \] (1)

**WHEEL** 
\[ \vec{v}_A = \vec{v}_D + \vec{v}_{AD} = \vec{\omega}_w \times \vec{r}_{AD} = \omega_{w}\vec{k} \times (0.95\vec{j}) = -0.95\omega_w\vec{i} \] (2)

\[ (1) = (2) \]

\[ \omega_w = -2 \text{ rad/s} \quad \omega_{AB} = 0 \]
\( \vec{a}_B = \vec{a}_C + \vec{a}_{B/C} = \ddot{\omega}_{BC} \times (\vec{\omega}_{BC} \times \vec{r}_{B/C}) + \vec{\alpha}_{BC} \times \vec{r}_{B/C} = -2\ddot{k} \times \left[ -2\ddot{k} \times (0.95\vec{j}) \right] + (-1.2\ddot{k}) \times (0.95\vec{j}) = 1.14\ddot{i} - 3.8\ddot{j} \)

**Member AB**

\[ \vec{a}_A = \vec{a}_B + \vec{a}_{A/B} = 1.14\ddot{i} - 3.8\ddot{j} + \alpha_{AB}\ddot{k} \times (-1.25\ddot{i}) = 1.14\ddot{i} - 3.8\ddot{j} - 1.25\alpha_{AB}\ddot{j} \quad (3) \]

**Wheel**

\[ \vec{a}_A = \vec{a}_O + \vec{a}_{A/O} = \alpha_w(0.5\ddot{i}) + (-2\ddot{k}) \times \left[ -2\ddot{k} \times (0.45\vec{j}) \right] + (-\alpha_w\ddot{k}) \times (0.45\vec{j}) = 0.5\alpha_w\ddot{i} - 1.8\ddot{j} + 0.45\alpha_w\ddot{i} \]

\[ (3) = (4) \quad \alpha_w = 1.2 \text{ rad} / \text{s}^2 \quad \alpha_{AB} = -1.6 \text{ rad} / \text{s}^2 \]

\[ \{ \omega_w = -2 \text{ rad} / \text{s}, \omega_{AB} = 0 \} \]
\[ \ddot{a}_{G1} = -2\kappa \times \left[ -2\kappa \times (0.475\,\vec{j}) \right] + (-1.2\kappa) \times (0.475\,\vec{j}) = 0.57\,\ddot{a}_x - 1.9\,\ddot{a}_y \]

**MEMBER BC**

\[ \bar{I}_{BC} = \frac{1}{12} m_{BC} l_{BC}^2 = \frac{1}{12} (1)(0.95)^2 = 0.0752 \text{ kg} \cdot \text{m}^2 \]

\[ \sum M_c = m_{BC} \ddot{a}_x d + \bar{I}_{BC} \alpha_{BC} \]

\[ B_x (0.95) = (1)(0.57)(0.475) + (0.0752)(1.2) \]

\[ B_x = 0.38 \text{ N} \]
\[ \bar{a}_{G2} = \bar{a}_B + \bar{a}_{G2}/B = 1.14\hat{i} - 3.8\hat{j} + (-1.6\hat{k}) \times (-0.625\hat{i}) = 1.14\hat{i} - 2.8\hat{j} \]

\[ \bar{a}_x = \begin{pmatrix} 1.14 \\ -2.8 \end{pmatrix} \]

\[ \bar{a}_y = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

\[ \vec{I}_{AB} \alpha_{AB} \]

\[ \sum F_x = m_{BC} \bar{a}_x \quad A_x - B_x = (2)(1.14) \quad \Rightarrow \quad A_x = 2.66 \text{ N} \]

\[ \bar{a}_{AB} = \frac{1}{12} m_{AB} \bar{a}_{x}^{2} = \frac{1}{12} (2)(1.25)^2 = 0.26 \text{ kg} \cdot \text{m}^2 \]

\[ \sum M_B = m_{AB} \bar{a}_d + \vec{I}_{AB} \alpha_{AB} \quad \Rightarrow \quad A_y (1.25) - (2)(9.81)(0.625) = -(2)(2.8)(0.625) + (0.26)(1.6) \]

\[ A_y = 7.34 \text{ N} \]

\[ \sum F_y = m_{BC} \bar{a}_y \]

\[ A_y - B_y - m_{AB}g = (2)(-2.8) \]

\[ 7.34 - B_y - (2)(9.81) = (2)(-2.8) \]

\[ \Rightarrow \quad B_y = -6.68 \text{ N} \]
\[
\vec{a}_o = \vec{a} = (1.2) (0.5) \vec{i} = 0.6 \vec{i}
\]

\[
\vec{I}_w = \frac{1}{2} m_w r_w^2 = \frac{1}{2} (5)(0.5)^2 = 0.625 \text{ kg} \cdot \text{m}^2
\]

**WHEEL**

**FBD**

- \[A_x, A_y\]
- \[m_w g\]
- \[P\]
- \[O, A, D\]

**KD**

\[
\begin{align*}
\sum M_D &= m_w \vec{a} d + \vec{I}_w \alpha_w \\
- A_x (0.95) + P (0.5) &= (5)(0.6)(0.5) + (0.625)(1.2) \\
P &= 9.554 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\sum F_x &= m_w \vec{a}_x \\
- A_x + P - F_f &= (5)(0.6) \\
F_f &= 3.894 \text{ N}
\end{align*}
\]

**KINETICS**

\[
\begin{align*}
\sum F_y &= 0 \\
- A_y + N - m_w g &= 0 \\
N &= 56.39 \text{ N}
\end{align*}
\]

\[
\mu = \frac{F_f}{N} = \frac{3.894}{56.39} = 0.069
\]
5. The unbalanced 20 kg wheel with the mass center at $G$ has a radius of gyration about $G$ of 202 mm. The wheel rolls down the 20° incline without slipping. In the position shown. The wheel has an angular velocity of 3 rad/s. Calculate the friction force $F$ acting on the wheel at this position.
SOLUTION

“General Motion”

**FBD**

\[
\begin{align*}
\mathbf{F}_{f} + \mathbf{N} &= \mathbf{m} \alpha \\
\mathbf{F}_{f} &= 2.617 \text{ N} \\
\mathbf{N} &= 160.971 \text{ N}
\end{align*}
\]

**KD**

\[
\bar{I} = m \bar{k}^2 = 20(0.202)^2 = 0.816 \text{ kgm}^2
\]

\[
\alpha = 15.597 \text{ rad} / \text{s}^2
\]

\[
\mathbf{a}_o = \alpha r = 0.25\alpha
\]

\[
\bar{a}_G = \bar{a}_O + \bar{a}_{G/O} = -0.25\alpha \bar{i} + \alpha \bar{k} \times (-0.075\bar{i}) + 3\bar{k} \times [3\bar{k} \times (-0.075\bar{i})]
\]

\[
\bar{a}_G = (-0.25\alpha + 0.675)\bar{i} - 0.075\bar{j}
\]

\[
\begin{align*}
(\sum F_x)_{ef} &= m\bar{a}_x \\
F_f + 5\alpha &= 80.604
\end{align*}
\]

\[
\begin{align*}
(\sum F_y)_{ef} &= m\bar{a}_y \\
N - mg \cos 20 &= 20(-0.075\alpha)
\end{align*}
\]

\[
\begin{align*}
N &= 184.367 - 1.5\alpha \\
(\sum M_G)_{ef} &= \bar{I}\alpha \\
N(0.075) + F_f(0.25) &= 0.816\alpha
\end{align*}
\]