Chapter 3: Differentiation

3.5 The Chain Rule and Parametric Equations

We know how to differentiate $y = f(u) = \sin u$ and $u = g(x) = x^2 - 4$, but how do we differentiate a composite like $F(x) = f(g(x)) = \sin (x^2 - 4)$? The differentiation formulas we have studied so far do not tell us how to calculate $F'(x)$. So how do we find the derivative of $F = f \circ g$? The answer is, with the Chain Rule, which says that the derivative of the composite of two differentiable functions is the product of their derivatives evaluated at appropriate points. The Chain Rule is one of the most important and widely used rules of differentiation. This section describes the rule and how to use it. We then apply the rule to describe curves in the plane and their tangent lines in another way.
Derivative of a Composite Function

We begin with examples.

**EXAMPLE 1** Relating Derivatives

The function \( y = \frac{3}{2} x = \frac{1}{2} (3x) \) is the composite of the functions \( y = \frac{1}{2} u \) and \( u = 3x \).

How are the derivatives of these functions related?

**Solution** We have

\[
\frac{dy}{dx} = \frac{3}{2}, \quad \frac{dy}{du} = \frac{1}{2}, \quad \text{and} \quad \frac{du}{dx} = 3.
\]

Since \( \frac{3}{2} = \frac{1}{2} \cdot 3 \), we see that

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.
\]

Is it an accident that

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.
\]

If we think of the derivative as a rate of change, our intuition allows us to see that this relationship is reasonable. If \( y = f(u) \) changes half as fast as \( u \) and \( u = g(x) \) changes three times as fast as \( x \), then we expect \( y \) to change 3/2 times as fast as \( x \). This effect is much like that of a multiple gear train (Figure 3.26).

**EXAMPLE 2**

The function

\[
y = 9x^4 + 6x^2 + 1 = (3x^2 + 1)^2
\]

is the composite of \( y = u^2 \) and \( u = 3x^2 + 1 \). Calculating derivatives, we see that

\[
\frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 6x = 2(3x^2 + 1) \cdot 6x = 36x^3 + 12x.
\]

Calculating the derivative from the expanded formula, we get

\[
\frac{dy}{dx} = \frac{d}{dx} \left( 9x^4 + 6x^2 + 1 \right) = 36x^3 + 12x.
\]

Once again,

\[
\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}.
\]

The derivative of the composite function \( f(g(x)) \) at \( x \) is the derivative of \( f \) at \( g(x) \) times the derivative of \( g \) at \( x \). This is known as the Chain Rule (Figure 3.27).
Intuitive “Proof” of the Chain Rule:
Let be the change in corresponding to a change of in .

Then the corresponding change in is

It would be tempting to write

and take the limit as :.

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x}
\]

\[
= \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}
\]

\[
= \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}
\]

(Note that as since is continuous.)

**THEOREM 3  The Chain Rule**
If is differentiable at the point and is differentiable at , then the composite function is differentiable at , and

In Leibniz’s notation, if and , then

where is evaluated at .

**Intuitive “Proof” of the Chain Rule:**
Let be the change in corresponding to a change of in , that is

Then the corresponding change in is

It would be tempting to write

and take the limit as :.

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x}
\]

\[
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\]

\[
= \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}
\]

(Note that as since is continuous.)
The only flaw in this reasoning is that in Equation (1) it might happen that $\Delta u = 0$ (even when $\Delta x \neq 0$) and, of course, we can’t divide by 0. The proof requires a different approach to overcome this flaw, and we give a precise proof in Section 3.8.

**EXAMPLE 3** Applying the Chain Rule

An object moves along the $x$-axis so that its position at any time $t \geq 0$ is given by $x(t) = \cos(t^2 + 1)$. Find the velocity of the object as a function of $t$.

**Solution** We know that the velocity is $dx/dt$. In this instance, $x$ is a composite function: $x = \cos(u)$ and $u = t^2 + 1$. We have

$$\frac{dx}{du} = -\sin(u) \quad x = \cos(u)$$

$$\frac{du}{dt} = 2t \quad u = t^2 + 1$$

By the Chain Rule,

$$\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = -\sin(u) \cdot 2t = -\sin(t^2 + 1) \cdot 2t = -2t \sin(t^2 + 1).$$

As we see from Example 3, a difficulty with the Leibniz notation is that it doesn’t state specifically where the derivatives are supposed to be evaluated.

**“Outside-Inside” Rule**

It sometimes helps to think about the Chain Rule this way: If $y = f(g(x))$, then

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x).$$

In words, differentiate the “outside” function $f$ and evaluate it at the “inside” function $g(x)$ left alone; then multiply by the derivative of the “inside function.”

**EXAMPLE 4** Differentiating from the Outside In

Differentiate $\sin(x^2 + x)$ with respect to $x$.

**Solution**

$$\frac{d}{dx} \sin(x^2 + x) = \cos(x^2 + x) \cdot (2x + 1)$$

**Repeated Use of the Chain Rule**

We sometimes have to use the Chain Rule two or more times to find a derivative. Here is an example.
EXAMPLE 5  A Three-Link “Chain”  
Find the derivative of \( g(t) = \tan(5 - \sin 2t) \).

Solution  Notice here that the tangent is a function of \( 5 - \sin 2t \), whereas the sine is a function of \( 2t \), which is itself a function of \( t \). Therefore, by the Chain Rule,

\[
g'(t) = \frac{d}{dt} \left( \tan(5 - \sin 2t) \right)
\]

\[
= \sec^2(5 - \sin 2t) \cdot \frac{d}{dt} (5 - \sin 2t)
\]

\[
= \sec^2(5 - \sin 2t) \cdot (0 - \cos 2t) \cdot \frac{d}{dt}(2t)
\]

\[
= \sec^2(5 - \sin 2t) \cdot (-\cos 2t) \cdot 2
\]

\[
= -2(\cos 2t) \sec^2(5 - \sin 2t).
\]

The Chain Rule with Powers of a Function

If \( f \) is a differentiable function of \( u \) and if \( u \) is a differentiable function of \( x \), then substituting \( y = f(u) \) into the Chain Rule formula

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

leads to the formula

\[
\frac{d}{dx} f(u) = f'(u) \frac{du}{dx}.
\]

Here’s an example of how it works: If \( n \) is a positive or negative integer and \( f(u) = u^n \), the Power Rules (Rules 2 and 7) tell us that \( f'(u) = nu^{n-1} \). If \( u \) is a differentiable function of \( x \), then we can use the Chain Rule to extend this to the Power Chain Rule:

\[
\frac{d}{dx} u^n = nu^{n-1} \frac{du}{dx}, \quad \frac{d}{du}(u^n) = nu^{n-1}
\]

EXAMPLE 6  Applying the Power Chain Rule

(a) \[
\frac{d}{dx} (5x^3 - x^4)^7 = 7(5x^3 - x^4)^6 \frac{d}{dx} (5x^3 - x^4)
\]

\[
= 7(5x^3 - x^4)^6(15x^2 - 4x^3)
\]

(b) \[
\frac{d}{dx} \left( \frac{1}{3x - 2} \right) = \frac{d}{dx}(3x - 2)^{-1}
\]

\[
= -1(3x - 2)^{-2} \frac{d}{dx}(3x - 2)
\]

\[
= -1(3x - 2)^{-2}(3)
\]

\[
= \frac{3}{(3x - 2)^2}
\]

In part (b) we could also have found the derivative with the Quotient Rule.
EXAMPLE 7 Finding Tangent Slopes

(a) Find the slope of the line tangent to the curve $y = \sin^3 x$ at the point where $x = \pi/3$.

(b) Show that the slope of every line tangent to the curve $y = 1/(1 - 2x)^3$ is positive.

Solution

(a) $\frac{dy}{dx} = 5 \sin^4 x \cdot \frac{d}{dx} \sin x$

$= 5 \sin^4 x \cos x$

The tangent line has slope

$\left. \frac{dy}{dx} \right|_{x=\pi/3} = 5 \left( \frac{\sqrt{3}}{2} \right)^4 \left( \frac{1}{2} \right) = \frac{45}{32}$.

(b) $\frac{dy}{dx} = \frac{d}{dx} (1 - 2x)^{-3}$

$= -3(1 - 2x)^{-4} \cdot \frac{d}{dx} (1 - 2x)$

$= -3(1 - 2x)^{-4} \cdot (-2)$

$= \frac{6}{(1 - 2x)^4}$

At any point $(x, y)$ on the curve, $x \neq 1/2$ and the slope of the tangent line is

$\frac{dy}{dx} = \frac{6}{(1 - 2x)^4}$,

the quotient of two positive numbers.

EXAMPLE 8 Radians Versus Degrees

It is important to remember that the formulas for the derivatives of both $\sin x$ and $\cos x$ were obtained under the assumption that $x$ is measured in radians, not degrees. The Chain Rule gives us new insight into the difference between the two. Since $180^\circ = \pi$ radians, $x^\circ = \pi x/180$ radians where $x^\circ$ means the angle $x$ measured in degrees.

By the Chain Rule,

$\frac{d}{dx} \sin(x^\circ) = \frac{d}{dx} \sin \left( \frac{\pi x}{180} \right) = \frac{\pi}{180} \cos \left( \frac{\pi x}{180} \right) = \frac{\pi}{180} \cos(x^\circ)$.

See Figure 3.28. Similarly, the derivative of $\cos(x^\circ)$ is $-(\pi/180) \sin(x^\circ)$.

The factor $\pi/180$, annoying in the first derivative, would compound with repeated differentiation. We see at a glance the compelling reason for the use of radian measure.

Parametric Equations

Instead of describing a curve by expressing the $y$-coordinate of a point $P(x, y)$ on the curve as a function of $x$, it is sometimes more convenient to describe the curve by expressing both coordinates as functions of a third variable $t$. Figure 3.29 shows the path of a moving particle described by a pair of equations, $x = f(t)$ and $y = g(t)$. For studying motion,
\( t \) usually denotes time. Equations like these are better than a Cartesian formula because they tell us the particle's position \((x, y) = (f(t), g(t))\) at any time \(t\).

**DEFINITION**  
**Parametric Curve**  
If \(x\) and \(y\) are given as functions over an interval of \(t\)-values, then the set of points defined by these equations is a **parametric curve**. The equations are **parametric equations** for the curve.

The variable \(t\) is a **parameter** for the curve, and its domain \(I\) is the **parameter interval**. If \(I\) is a closed interval, \(a \leq t \leq b\), the point \((f(a), g(a))\) is the **initial point** of the curve. The point \((f(b), g(b))\) is the **terminal point**. When we give parametric equations and a parameter interval for a curve, we say that we have **parametrized** the curve. The equations and interval together constitute a **parametrization** of the curve.

**EXAMPLE 9**  
**Moving Counterclockwise on a Circle**

Graph the parametric curves

- (a) \(x = \cos t, \quad y = \sin t, \quad 0 \leq t \leq 2\pi\).
- (b) \(x = a \cos t, \quad y = a \sin t, \quad 0 \leq t \leq 2\pi\).

**Solution**

- (a) Since \(x^2 + y^2 = \cos^2 t + \sin^2 t = 1\), the parametric curve lies along the unit circle \(x^2 + y^2 = 1\). As \(t\) increases from 0 to \(2\pi\), the point \((x, y) = (\cos t, \sin t)\) starts at \((1, 0)\) and traces the entire circle once counterclockwise (Figure 3.30).
- (b) For \(x = a \cos t, y = a \sin t, 0 \leq t \leq 2\pi\), we have \(x^2 + y^2 = a^2 \cos^2 t + a^2 \sin^2 t = a^2\). The parametrization describes a motion that begins at the point \((a, 0)\) and traverses the circle \(x^2 + y^2 = a^2\) once counterclockwise, returning to \((a, 0)\) at \(t = 2\pi\).  

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EXAMPLE 10 Moving Along a Parabola

The position \( P(x, y) \) of a particle moving in the \( xy \)-plane is given by the equations and parameter interval

\[
x = \sqrt{t}, \quad y = t, \quad t \geq 0.
\]

Identify the path traced by the particle and describe the motion.

Solution We try to identify the path by eliminating \( t \) between the equations and

With any luck, this will produce a recognizable algebraic relation between \( x \) and \( y \).

We find that

\[ y = t = (\sqrt{t})^2 = x^2. \]

Thus, the particle’s position coordinates satisfy the equation \( y = x^2 \), so the particle moves along the parabola \( y = x^2 \).

It would be a mistake, however, to conclude that the particle’s path is the entire parabola \( y = x^2 \); it is only half the parabola. The particle’s \( x \)-coordinate is never negative. The particle starts at \((0, 0)\) when \( t = 0 \) and rises into the first quadrant as \( t \) increases (Figure 3.31). The parameter interval is \([0, \infty)\) and there is no terminal point.

EXAMPLE 11 Parametrizing a Line Segment

Find a parametrization for the line segment with endpoints \((-2, 1)\) and \((3, 5)\).

Solution Using \((-2, 1)\) we create the parametric equations

\[
x = -2 + at, \quad y = 1 + bt.
\]

These represent a line, as we can see by solving each equation for \( t \) and equating to obtain

\[
\frac{x + 2}{a} = \frac{y - 1}{b}.
\]

This line goes through the point \((-2, 1)\) when \( t = 0 \). We determine \( a \) and \( b \) so that the line goes through \((3, 5)\) when \( t = 1 \).

\[
\begin{align*}
3 &= -2 + a \quad \Rightarrow \quad a = 5, \quad x = 3 \text{ when } t = 1. \\
5 &= 1 + b \quad \Rightarrow \quad b = 4, \quad y = 5 \text{ when } t = 1.
\end{align*}
\]

Therefore,

\[
x = -2 + 5t, \quad y = 1 + 4t, \quad 0 \leq t \leq 1
\]

is a parametrization of the line segment with initial point \((-2, 1)\) and terminal point \((3, 5)\).

Slopes of Parametrized Curves

A parametrized curve \( x = f(t) \) and \( y = g(t) \) is differentiable at \( t \) if \( f \) and \( g \) are differentiable at \( t \). At a point on a differentiable parametrized curve where \( y \) is also a differentiable function of \( x \), the derivatives \( \frac{dy}{dt}, \frac{dx}{dt}, \) and \( \frac{dy}{dx} \) are related by the Chain Rule:

\[
\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}.
\]

If \( \frac{dx}{dt} \neq 0 \), we may divide both sides of this equation by \( \frac{dx}{dt} \) to solve for \( \frac{dy}{dx} \).
EXAMPLE 12  Differentiating with a Parameter

If \( x = 2t + 3 \) and \( y = t^2 - 1 \), find the value of \( \frac{dy}{dx} \) at \( t = 6 \).

Solution  Equation (2) gives \( \frac{dy}{dx} \) as a function of \( t \):

\[
\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{2} = t = \frac{x - 3}{2}.
\]

When \( t = 6 \), \( \frac{dy}{dx} = 6 \). Notice that we are also able to find the derivative \( \frac{dy}{dx} \) as a function of \( x \).

EXAMPLE 13  Moving Along the Ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)

Describe the motion of a particle whose position \( P(x, y) \) at time \( t \) is given by

\[
x = a \cos t, \quad y = b \sin t, \quad 0 \leq t \leq 2\pi.
\]

Find the line tangent to the curve at the point \( \left( \frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}} \right) \), where \( t = \pi/4 \). (The constants \( a \) and \( b \) are both positive.)

Solution  We find a Cartesian equation for the particle’s coordinates by eliminating \( t \) between the equations

\[
\cos t = \frac{x}{a}, \quad \sin t = \frac{y}{b}.
\]

The identity \( \cos^2 t + \sin^2 t = 1 \), yields

\[
\left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1, \quad \text{or} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]

The particle’s coordinates \( (x, y) \) satisfy the equation \( \left( \frac{x^2}{a^2} \right) + \left( \frac{y^2}{b^2} \right) = 1 \), so the particle moves along this ellipse. When \( t = 0 \), the particle’s coordinates are

\[
x = a \cos(0) = a, \quad y = b \sin(0) = 0,
\]

so the motion starts at \( (a, 0) \). As \( t \) increases, the particle rises and moves toward the left, moving counterclockwise. It traverses the ellipse once, returning to its starting position \( (a, 0) \) at \( t = 2\pi \).

The slope of the tangent line to the ellipse when \( t = \pi/4 \) is

\[
\frac{dy}{dx} \bigg|_{t=\pi/4} = \frac{dy/dt}{dx/dt} \bigg|_{t=\pi/4} = \frac{b \cos t}{-a \sin t} \bigg|_{t=\pi/4} = \frac{b/\sqrt{2}}{-a/\sqrt{2}} = -\frac{b}{a}.
\]
The tangent line is
\[ y - \frac{b}{\sqrt{2}} = -\frac{b}{a} \left( x - \frac{a}{\sqrt{2}} \right) \]
or
\[ y = \frac{b}{\sqrt{2}} - \frac{b}{a} \left( x - \frac{a}{\sqrt{2}} \right) \]

or
\[ y = -\frac{b}{a} x + \sqrt{2}b. \]

If parametric equations define \( y \) as a twice-differentiable function of \( x \), we can apply Equation (2) to the function to calculate \( \frac{d^2y}{dx^2} \) as a function of \( t \):

\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dt} \right) = \frac{dy'/dt}{dx/dt}. \quad \text{Eq. (2) with } y' \text{ in place of } y \]

**Example 14** Finding \( \frac{d^2y}{dx^2} \) for a Parametrized Curve

Find \( \frac{d^2y}{dx^2} \) as a function of \( t \) if \( x = t - t^2, \ y = t - t^3 \).

**Solution**

1. Express \( y' = \frac{dy}{dx} \) in terms of \( t \).
2. Find \( \frac{dy'}{dt} \).
3. Divide \( \frac{dy'}{dt} \) by \( \frac{dx}{dt} \).

\[ y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 - 3t^2}{1 - 2t} \]

2. Differentiate \( y' \) with respect to \( t \).

\[ \frac{dy'}{dt} = \frac{d}{dt} \left( \frac{1 - 3t^2}{1 - 2t} \right) = \frac{2 - 6t + 6t^2}{(1 - 2t)^2} \]

3. Divide \( \frac{dy'}{dt} \) by \( \frac{dx}{dt} \).

\[ \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{(2 - 6t + 6t^2)/(1 - 2t)^2}{1 - 2t} = \frac{2 - 6t + 6t^2}{(1 - 2t)^3} \]

**Example 15** Dropping Emergency Supplies

A Red Cross aircraft is dropping emergency food and medical supplies into a disaster area. If the aircraft releases the supplies immediately above the edge of an open field 700 ft long and if the cargo moves along the path

\[ x = 120t \quad \text{and} \quad y = -16t^2 + 500, \quad t \geq 0 \]
does the cargo land in the field? The coordinates $x$ and $y$ are measured in feet, and the parameter $t$ (time since release) in seconds. Find a Cartesian equation for the path of the falling cargo (Figure 3.32) and the cargo’s rate of descent relative to its forward motion when it hits the ground.

**Solution** The cargo hits the ground when which occurs at time $t$ when

$$-16t^2 + 500 = 0$$

Set $y = 0$.

$$t = \frac{\sqrt{500}}{16} = \frac{5\sqrt{5}}{2} \text{ sec.}$$

The $x$-coordinate at the time of the release is $x = 0$. At the time the cargo hits the ground, the $x$-coordinate is

$$x = 120t = 120\left(\frac{5\sqrt{5}}{2}\right) = 300\sqrt{5} \text{ ft.}$$

Since $300\sqrt{5} \approx 670.8 < 700$, the cargo does land in the field.

We find a Cartesian equation for the cargo’s coordinates by eliminating $t$ between the parametric equations:

$$y = -16t^2 + 500 \quad \text{Parametric equation for } y$$

Substitute for $t$ from the equation $x = 120t$.

$$= -16\left(\frac{x}{120}\right)^2 + 500$$

$$= -\frac{1}{900}x^2 + 500. \quad \text{A parabola}$$

The rate of descent relative to its forward motion when the cargo hits the ground is

$$\left.\frac{dy}{dx}\right|_{t=5\sqrt{5}/2} = \frac{dy/dt}{dx/dt}\bigg|_{t=5\sqrt{5}/2}$$

$$= \frac{-32t}{120}\bigg|_{t=5\sqrt{5}/2}$$

$$= -\frac{2\sqrt{5}}{3} \approx -1.49.$$ 

Thus, it is falling about 1.5 feet for every foot of forward motion when it hits the ground.

**USING TECHNOLOGY** Simulation of Motion on a Vertical Line

The parametric equations

$$x(t) = c, \quad y(t) = f(t)$$

will illuminate pixels along the vertical line $x = c$. If $f(t)$ denotes the height of a moving body at time $t$, graphing $(x(t), y(t)) = (c, f(t))$ will simulate the actual motion. Try it for the rock in Example 5, Section 3.3 with $x(t) = 2$, say, and $y(t) = 160t - 16t^2$, in dot mode with $\text{Step} = 0.1$. Why does the spacing of the dots vary? Why does the grapher seem to stop after it reaches the top? (Try the plots for $0 \leq t \leq 5$ and $5 \leq t \leq 10$ separately.)

For a second experiment, plot the parametric equations

$$x(t) = t, \quad y(t) = 160t - 16t^2$$
together with the vertical line simulation of the motion, again in dot mode. Use what you know about the behavior of the rock from the calculations of Example 5 to select a window size that will display all the interesting behavior.

Standard Parametrizations and Derivative Rules

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